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**Mathematical Foundations
of Social Anthropology**

Mouton · Paris · The Hague

ACKNOWLEDGEMENTS

This work has been long in preparation, and the number of people who have read and commented on parts of it are numerous, as are those whose direct or indirect support made it possible. At different times, a great number of mathematicians or mathematically oriented people have read all or part of the present work and given their comments. Only in one case (that of Tom Duchamp) did any collaboration result, and this is clearly acknowledged where appropriate. Among others who should be thanked for their lively and invigorating comments or assistance are: Klaus Witz, Francois Terrain, Harrison White, Dwight Read, Richard Greechie, L. L. Cavalli-Sforza, W. J. Schull, Masatoshi Nei, John Lombardi, Kris Lehman, John Atkins, Philip Newman, B. J. Williams, and Eugene Giles. Gomer Thomas and Peter Wollin provided detailed suggestions on the final manuscript. Clearly, none of these people are responsible for any of the present content, and any whom I have, by omission, neglected to thank, are given my apology and appreciation.

Support for production of this work was largely personal. However, at different times, assistance has been provided by the following institutions: University of Illinois, Departments of Mathematics and Anthropology; Champaign County (Illinois) Department of Public Welfare; University of Texas Health Science Center at Houston; University of Washington; Harvard University Department of Social Relations; Southern Illinois University; University of California, Los Angeles; Rice University. I wish to give my grateful appreciation to the many technical typists who have made this work readable over the years, and especially to the Maison des Sciences de l'Homme which has seen fit to publish it. I give my personal gratitude to Clemens Heller of the Maison for his interest and encouraging support. The hard typing the final manuscript was done by Sandra Stasny.

ERRATA

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page 49 line 9 R^{j+1} for any positive integer n should read R^{j+k} for any positive integer k

page 81

line 3 $+ 1/10$ should read $+ 1/100$

line 4 $+ 10\%$ of L should read $+ 5\%$ of L

line 8 10 times fewer should read 100 times fewer

page 57

second half, following the words "Thus if", and page 58, first half, including matrices M_1 and M_2 , should go to page 59, between matrices S, R^1, L^1 , and paragraph starting with "Notice that either..."

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reference to Ballouff, P. A., 1974a: title should read Mathematical Models of Social and Cognitive Structures, Urbana: University of Illinois Press.

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PREFACE

Objective

Over the last several decades, at an increasing pace, social anthropology has witnessed mathematical formulations of its two most prominent branches: kinship and marriage. While not all anthropologists have welcomed intervention of "formal" methods, it is now clear that in any case they are here to stay. In the same period we have also seen the development of formal, if not mathematical, treatment of myth, folktales, riddles, and other materials which form the core of cultural semantics.

Of these two major developments, the present author has largely been a contri- butor to the first, not the second, but he has also found that there are numerous problems, principally of interpretation, left unanswered and indeed unasked by the "pure" theory of marriage and kinship. On the other hand, there are quite a number of questions answered by the pure theory without reference to semiotic concepts.

In the present work we therefore set out after this objective: to outline the pure theory of marriage and kinship (which we shall call "marriage theory" for sim- plicity), to establish the independent existence of this field in the jungle of demographic and mathematical models, and to show its surprising potential for re- sponding to some very critical questions. In a companion work, we will provide sufficient development of cultural semantic theory so that one may see both that many of the claims and methods of the less mathematical "structuralists" are indeed well justified, and to show that the fundamental structures which motivate marriage theory (namely, marriage rules) do indeed have a proper foundation in cultural theory. (In a sense, we hereby carry out the program of C. Levy-Strauss in Chapter 15 of Structural Anthropology.)

Philosophy

There is an important overlying philosophy of this work, reflected in the above division into two major topics, but also to be noted in marriage theory itself on several occasions. This philosophy may be stated as follows: the accessible re- sults are limited by the available techniques. While this may seem trite, it appears to be a hard lesson. For example, we discuss early in the work that the major techniques of demography have limits of applicability well known to demogra- phers. We are then able to gain results in marriage theory in the precise area of greatest weakness of demography. Similarly, we are able to at least fill one (small but) important gap in genetic theory. Likewise, when we abandon marriage theory to

finally enter cultural theory, we shall also abandon nearly all the mathematics of the work, save only for the necessities imposed by the rules of mathematical syntax. With a few byways on the path, we shall then arrive at a formal conceptualization of the ideas used intuitively in this work.

The present book is a text on theory, not history, and as such it contains only as much bibliography as necessary to develop the theory. This position is also justified by the fact that I have elsewhere published materials of historical nature (Ballouff, 1974b, 1974c). On the other hand, to motivate or interpret the theoretical developments, it is often necessary to present short sections of an essentially philosophical nature. I hope the reader may find these helpful.

Readers Background

A comment on the presumed background of the reader is necessary. While there is certainly a necessity for presentation of results in forms readable to the untrained, to those trained, there is also an obvious need to present the result in the first place. Whatever may stand the first test then deserves the second, and since this is the first time (to the author's knowledge) that all aspects of the present problem have been treated in a single place from the same perspective, we admit from the outset its experimental nature. We shall worry about "popularization", if at all, only when it is clear that there is something to popularize.

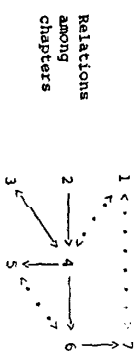
Marriage Theory as Biological Theory

While one section of this text will discuss certain contributions to genetics, in general, the reader with advanced knowledge of genetics or demography will find the present approach unusual. For example, in the "demographic" sections, we find particular family sizes represented as "equilibrium sizes" yet at the same time, we have no information on "age structure" of the population, so critical in the usual demographic mathematics. These same family size arguments likewise have nothing whatever to do with "selection" in the sense of the usual population genetics theory, since the very models which induce allocations of individuals by family, allow for the possibility that biological reproduction is random among mates. It is for these reasons that the present theory is not titled "Foundations of Population Biology" or the like. The present perspective is strictly social anthropological - if the results have implication for biology, it is only a fair return on the many insights that have crossed the boundary in the other direction.

HOW TO READ THIS WORK

Chapters are interrelated as shown in the chart below, using three symbols:

- strong dependence for concepts or development, should be read in direction of the arrow.
- ↔ strong relation in content, may be read in either order.
- ◀...▶ related contents, order not important.



Thus, one may find the "real meat" of the book by reading section 2.1, then chapters 4, 6, and 7. Chapters 5 and 6 are alternative approaches to the same or a similar problem, and in fact the results of 6 could also be done in terms of the results of 5. Chapter 7 is more or less "philosophical" showing what might well be done with the scheme presented, while chapter 1 is "philosophical" in that it motivates the work by showing its existence (as a theorem) and its distinction from other fields of study. Chapter 3, like chapter 5, shows one path that could be followed more extensively in future work.

Notice that this interrelationship of materials and short discussion suggests that in one sense this text is a treatment of the possible ways of working on "the marriage problem". Reading in the order of the chapter numbers will likely help perceive the survey from the more common to the least common methods, while reading "only for the meat" one might plunge directly into section 6.4. The Synopsis should help a reader decide his approach.