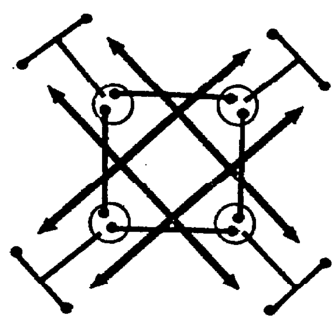
CHAPTER 2

THE EXISTENCE OF MINIMAL STRUCTURES

Introduction

This chapter summarizes origins of the most basic ideas of the minimal structure theory. This summary is illustrative, not exhaustive, as there are probably at least hundreds of possible anthropological or ethnographic literature citations which have in one or another form used minimal structures. One origin may be traced to genetist Sewall Wright (1921), showing that the maintenance of systems of (genetic) inbreeding can be associated with a smallest number of individuals required in each generation to maintain the system.* To do this, Wright assumed a rather ideal case of two offspring to each mating, and that these were always one male and one female. These restrictions allowed him to calculate, for example that where mating to first cousins is prohibited, at least eight people are required to represent each generation.

Using a slight variant of the ordinary diagramming technique, this rule may be shown as:



In this picture, I use a dot "•" to represent a person, whether male or female, a line between two dots "•——•" to show that the two are siblings, a circle around two dots to show a "marriage" between them, and a sibling line with a vertical insert into a marriage circle "(:)—|" to show descent from a marriage. The double pointed arrows show available partners for marriage under the proposed mating convention. Hence, in the above diagram, only those partners across the diagonal of the square are available to each other.

---

* In chapter 5 I discuss in much more detail relations between population genetic theory (evolutionary theory) and minimal structure theory. In particular, I do <u>not</u> assume that social ascription is identical to biological inheritance. I refer to Wright here because his was an early and systematic use of a minimally structured system in a mathematical argument.

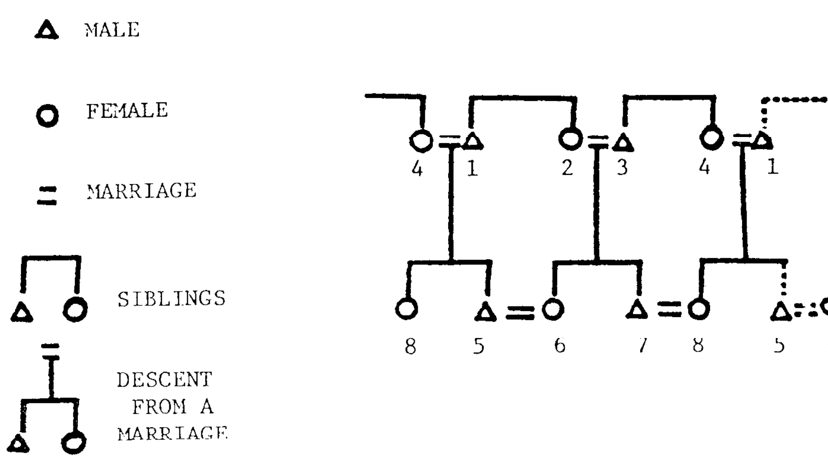
Call the number of "distinct families" (i.e., the number of groups of persons connected by the "sibling" link) required in each generation to maintain the stability of the rule, the minimal stability number for that system. It is very useful to find such a diagram for each marriage system one wishes to study, a procedure already followed by many ethnographers and ethnologists. For example, R. T. Zuidema has constructed precisely such a system, with seven sides and from four to five layers of generation depth, in order to discuss North American clan system (Zuidema, 1964, 1965): E. A. Cook presented similar arguments for the Manga of New Guinea (1967, 1969): B. Ruhemann has shown similar properties of the classic Australian systems (1945). One property of these same diagrams is that one may study them more easily as patterns of cycles and chains of an abstract mathematical system. F. B. Livingstone (1959, 1969) and C. Levi-Strauss (1961, Chapter 15; 1969) have both indicated this.

Next, it has long since been pointed out that related properties of marriage systems may be represented as group operators of various sorts. Andre Weil presented the needed demonstration in the appendix to the 1949 edition of Levi-Strauss (1969) referenced above, subsequently reprinted in H. White (1963). This later book is also important for its axiomatization of kinship systems, and its mapping of such systems onto role trees suitable for other analysis. (I have also used operators to represent certain properties of marriage systems; see Ballonoff, 1976b.)

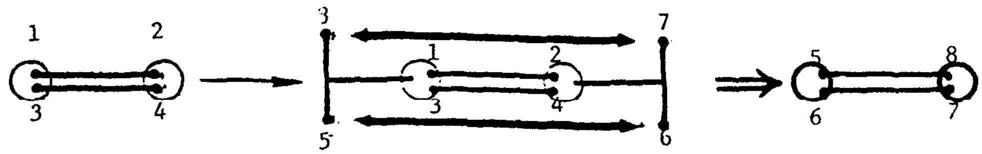
The purpose of the following is to demonstrate by example several minimal structures, selected to show the range of application. At the same time, I shall illustrate my notation.

Double First Cousin Mating

This system is one in which each individual of a marriage pair is a first cousin (but not closer relative) of the individual to whom he or she is paired. A typical notation in "geneological style" is shown below, where the key on the left shows the typical social-anthropological meanings of the symbols:



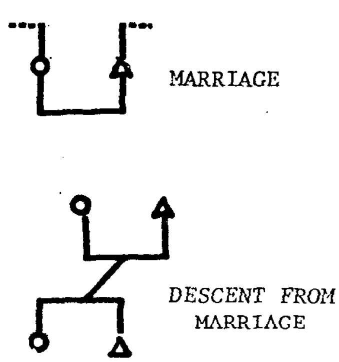
Individuals 1, 2, 3, and 4 are all of the same generation, while 5, 6, 7, and 8 are their offspring. Clearly, 7 and 8 are first cousins of each other through both their father (partrilineage) and mother (matrilineage) in the ordinary sense of "first-cousin". The same is true of 5 and 6. This system thus can continue to reproduce first cousin mating so long as the rather idealized assumptions used to draw the diagram are met in reality. A representation of this system in my universal notation is shown below. Note that an individuals' sex is not indicated in the diagram, since each mating presumably must have at least one of each sex, and the different permutations of sex indices are not important to diagramming this example.

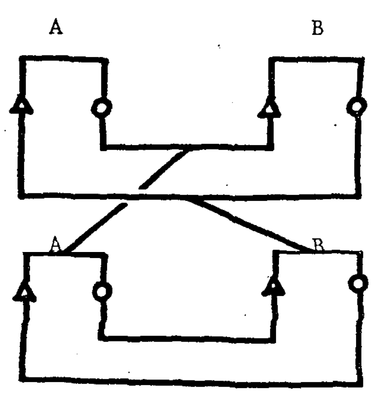


Thus, the configuration which initiated this particular sequence (i.e., that which has individuals 1, 2, 3, 4) is a stable self-reproducing "minimal structure" except for the labeling of individual names on the various instances of the structure. No smaller such structure can represent this operation of a "double first cousin" prescriptive rule.

<u>Dual Society in Sister Exchange</u>

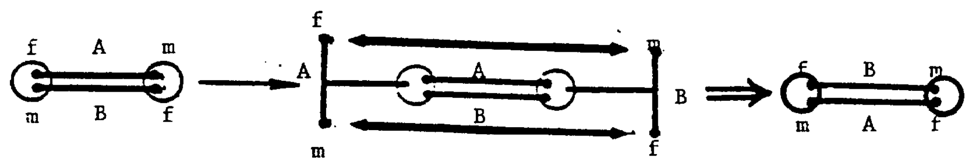
This example is taken from Fox (1967:180), a standard text for marriage theory in anthropology. A society is divided into two large divisions (moieties) and each family in the society belongs to one or the other of these. Male members of moiety must take their wives from families in the other moiety. Fox labels moieties as "A" and "B", and diagrams this system as shown below:





In this particular diagram, offspring of a mating are assigned to the moiety of their mother.

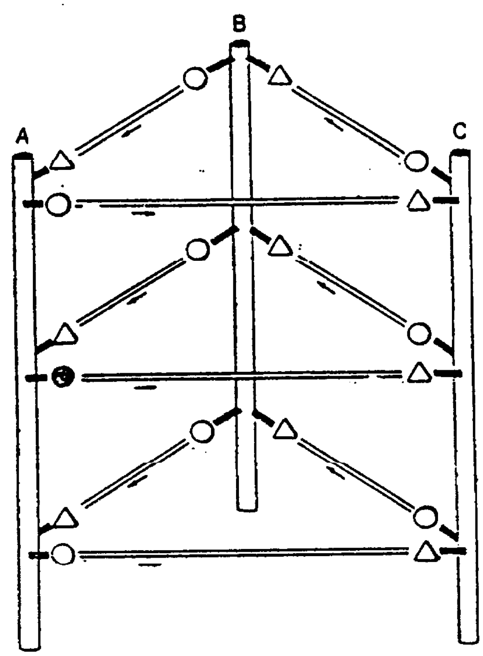
A diagram in universal notation for this system is shown below. In this example, I have labeled the sexes with small letters "m" and "f", to keep track of descent lines. The letters "A" and "B" next to a sibling pair show moiety affiliation:



Note that the self-reproducing "minimal structure" in this example is isomorphic to the minimal structure of example 1, except for the labelings.

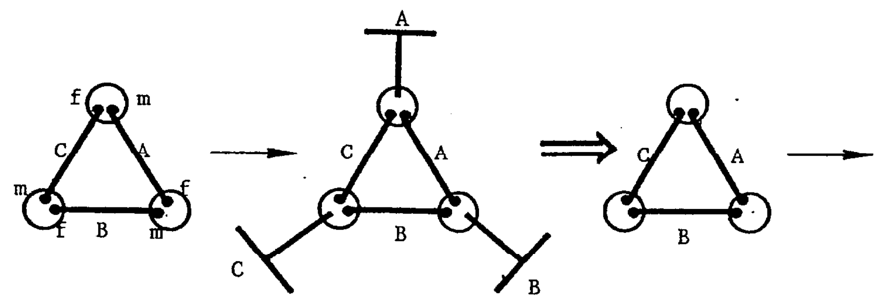
Patrilateral, Patrilineal Connubial Alliance

The source of this example is M. Harris (1971:368). It represents one variant of the "crow-omaha" systems of kin labeling. This particular source is the only one using something close to my "universal system" and may be viewed for additional examples of this and other systems. In the representation below, I have removed the kinship terminologies from Harris' diagram, so that the underlying "marriage" structure is more apparent:



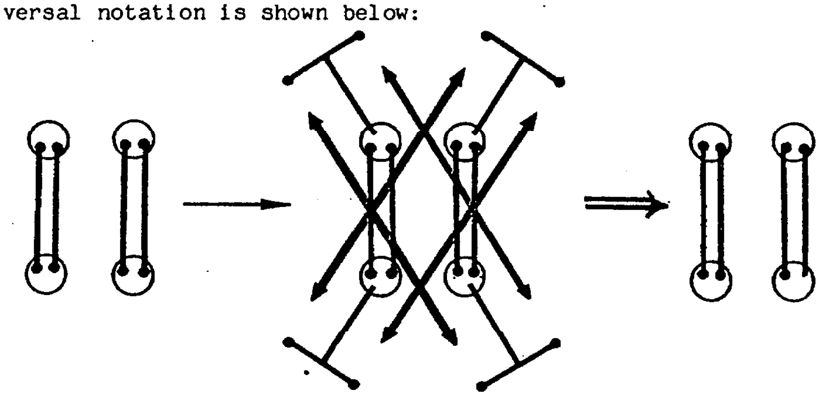
from Harris, Culture Man and Nature, 1971, page 368

The minimal structure for this system is a "triangle", as shown below, in a reproducing sequence. I note that this structure is isomorphic to structures for other "Crow-omaha" systems; to systems of "forced cross-cousin marriage" with lineage avoidance; and to systems of circulating marriage which have three labeled "clans" that must be maintained.

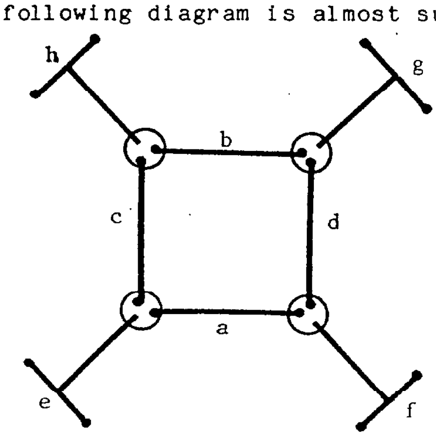


"American" Marriage Rule

The "American" marriage rule may be constructed as follows: One may not marry a brother or sister or first cousin, and marries within ones own generation. A system of four reproducing families is needed to maintain this system. A representation of this in universal notation is shown below:



Now construct a diagram in order to determine the minimal configuration for "American" marriage. In order to have the vocabulary, I will call the set of individuals related by a sibling line a "distinct family", a "sibling group", or a "sibship" interchangeably. Notice, however, that "distinct family" does not include the "parents" of a sibship. To simplify illustrative matters a bit, I presume two individuals of different sex in each distinct family, as is done in ordinary social anthropology diagrams. I shall argue that the following diagram is almost sufficient to explain "American" on a blackboard:



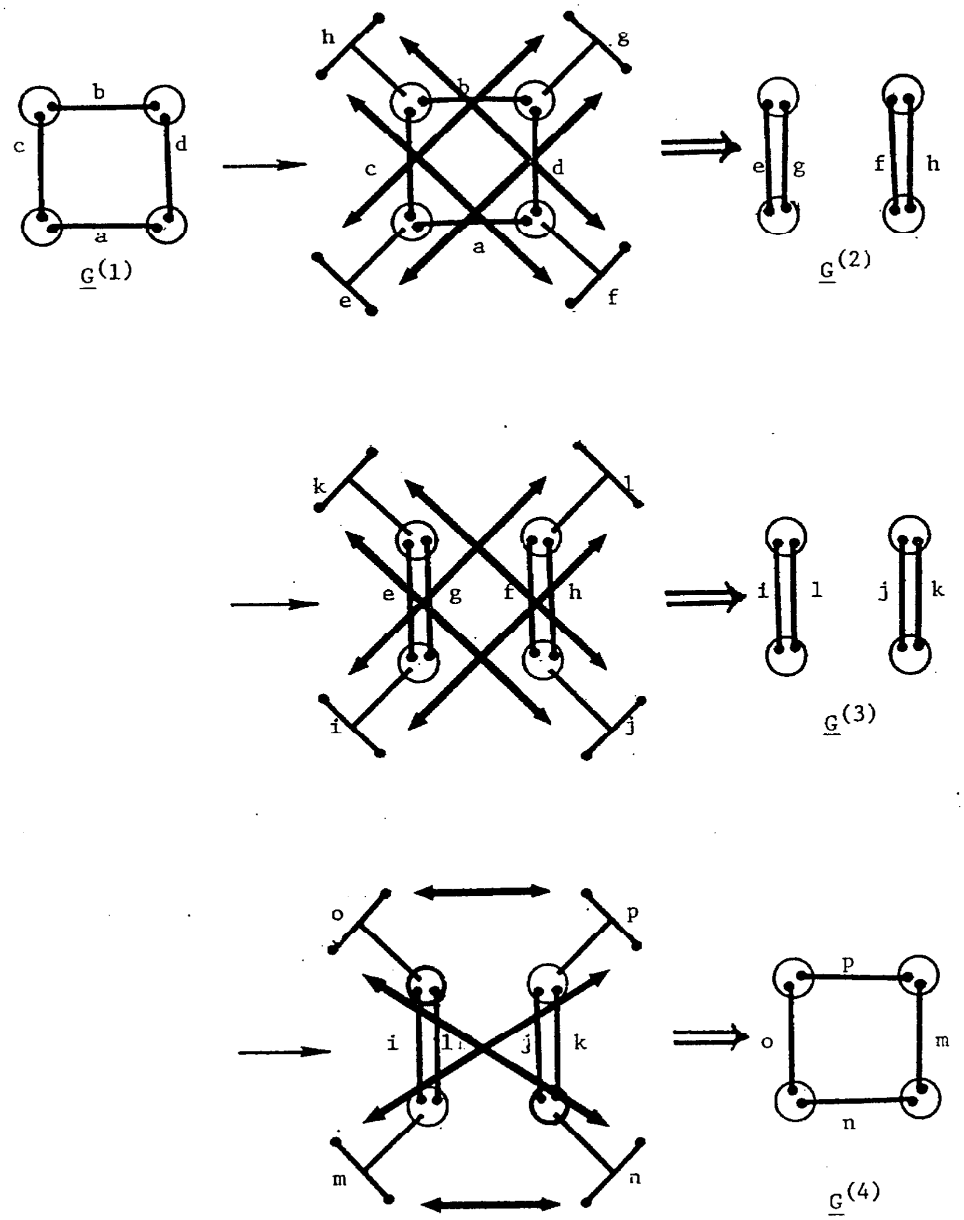
The small letters a to d label distinct families of the "older" generation, while e to h label the "younger" generation. I shall reason from distinct family e, but this choice is arbitrary. Sibship e is descended from mating of members of distinct families a and c. This means that members of e will consider people of h and f to be their "first cousins". A member of e has gone out to seek a mate. He is prevented by his concepts from mating with anyone in the older generation, or with the other member of his sibship, or with his first cousins in h and f. He thus has a seeming forced choice of a partner from sibship g.

A diagram with four distinct families per generation is the smallest diagram which can describe this system, and four is the minimal stability number for "American". However, in the smallest stable system for "American", either of the following two verbal descriptions are adequate: 1) an American will always avoid his first cousin as a marriage partner, or 2) an American (in such a small society) is always forced to marry his second cousin. Thus if an ethnographer were to walk into a small group of Americans stranded on an island for the last 100 years, it might be difficult to decide which system they "really" used.

I call this sort of situation the "dilemma of ethnographic choice". It is particularly important "in the real world" for small systems, and in this particular theory for the adequate description of the smallest stable system. Nor is the example above the only way in which the dilemma may occur. Suppose we carry out the indicated marriages in the diagram for American, allow reproduction, and try to form a new cycle. Use a double pointed arrow <——> to indicate possible marriages, a short double shafted arrow ===> to show the result of indicated marriages, and a single shafted arrow ——> to mean "after reproduction in the next generation".

In each case I have prohibited only marriages to first cousins or siblings. In generation one, labeled $\underline{G}^{(1)}$, he observes what appears to be a closed cycle of distinct families. These reproduce, and their offspring marry by either prescribed choice of second cousin or by a proscribed exclusion of siblings and first cousins. The resulting configuration in $\underline{G}^{(2)}$ could easily be described as a "dual clan" system, and the mating pattern might be said to be "clan exogamy", or "prescribed marriage into the clan of the matrilineal uncle", or any one of several other descriptions. In generation three the mating pattern seems to remain "dual clan exogamy", but then in $\underline{G}^{(4)}$ the system "shifts back" to "all first cousin exclusion".

I do not attempt to assert which of the <u>verbal</u> descriptions is the "correct" one - in a real situation, that is a job for ethnographic semantics. What is important here about any of the descriptions is that each requires four distinct families to "make the system work", so that the minimal stability number remains four no matter which <u>verbal</u> explanation is given. (In fact, from Appendix I, the unique minimal configuration for "American" is not the "square", but is instead the "two rectangles".)



Whenever such numbers can be found for a marriage system, call it a "minimal stability number" of the system in question. A minimal stability number will describe the least number of exogamous units needed to describe a marriage system that also preserves the logic of the relationships between exogamous units. It will not, however, summarize all of the conditions on reproduction needed to make a system biologically stable. (This is approached later in the text, as well as in Ballonoff, 1976 chapters 5 and 6, or Ballonoff, 1982, and in Appendix 1 here). Similar reasoning was used by E. A. Cook (1967:225-227) to construct a model of the marriage system of the Manga of New Guinea. Louis Dumont (1966) described "asymmetrical intermarriage in an eight-section system" using a more ad hoc notation. Dumont used the arrows to show direction of giving of women into marriage, from groups related by heavy black lines. Dumont's diagrams can be redrawn and shown to be equivalent to redrawing the square configuration found in the earlier study of "American".

Comparative Examples

In a manner similar to the last few paragraphs, one could demonstrate numerous examples of equivalences between diagrams existing in ethnographic literature, and graphs generated by my system. For example, triangular drawings for "cross cousin" systems appear in several places, such as Fox (1967:204) and Levi-Strauss (1969:126). Numerous variants of section systems are discussed in Elkin (1950) and Livingstone (1959).

However, one author has been quite thorough in discussion of "classificatory relationship systems". The 1945 article of B. Ruhemann is of interest because it approached the problem of marriage systems empirically, rather than axiomatically, yet agrees with my results on major points. For example, Ruhemann says ". . . a multiplicity of different combinations of descriptive terms may all denote one and the same classificatory relationship . . ." (1945:533). In some of the extended examples above, only a re-labeling of items was needed to show equivalence. Similarly, it was seen in one of the square diagrams that the statement "a person must always marry their second cousin in a closed, four clan society", can be equivalent to: "one may never marry a sibling or first cousin" in the diagram that represents the rule. Therefore, both an empirical and an axiomatic presentation of kinship can lead to the conclusion that "the same system" may be described in numerous ways. This was described above as the "dilemma of ethnographic choice".

Further on, Ruhemann presents several "guiding principles" for construction of adequate descriptions. Two of these principles are:

"(1) The system must be self contained and consistent . . ."

"(2) The system must be able to reproduce itself after a certain number of generations . . ." (1945:543)

Principles 1 and 2 are nearly equivalent to the starting condition for my analysis: This condition was called "minimal stability".

Since Ruhemann dedicated many pages to demonstrating the adequacy of her derived principles, it is not surprising to find that her succeeding pages all contain diagrams equivalent to those used here. Ruhemann's diagram on her page 546 is equivalent to one of my squares, that on her page 547 to a triangle. On her page 548, a six section system was presented which does not have a corresponding diagram in this chapter. However, the presentation of Ruhemann in that case is in the form of a "group table" and since an interpretation of such work in terms of group theory of mathematics is possible (White, 1963; Weil, 1963) credit may be given there for showing that an equivalent diagram is possible. This same comment holds for

Ruhemann's diagrams on page 560, where a five section system was represented. On page 567, a six section system was represented but this may be shown to be equivalent to a three sided diagram. Some of Ruhemann's results are summarized at the end of this chapter.

Summary of Major Examples

Two kinds of numbers commonly occur in the examples: the number of reproducing lines needed to represent the minimal structure of a mating pattern or marriage rules; and the number of "levels of inclusion" needed to specify the basic units to which a rule applies. I shall refer to the number of units in the minimal structure as the structural numbers, and to the number of layers of inclusion as sequence numbers (denoted later with Roman numerals, as "Sequence I", etc.).

Calling $|\underline{B}|$ = the number of distinct families per generation; $|\underline{M}|$ = the number of marriage pairs; $|m|$ = the number of males; $|f|$ = the number of females; and g = the total population of the generation; it can be shown (from the Appendix I here or from Ballonoff, 1976, Chapter 4) that $|\underline{B}| \leq |\underline{M}| = |m| = |f| = (1/2)g$. Since for most systems $|\underline{B}| = |\underline{M}|$ as well, one may speak of a unique structural number s characteristic of each marriage rule. As defined in Appendix I, this structural number s is the number $|\underline{M}|$ of marriages in the minimal structure. As was apparent from even the earliest attempts at quantifying "inbreeding" (Pearl, 1913) such numbers are quite important in the various population statistics of a system. Elaboration of this particular point for Australian systems may be found in Yengoyan (1968) or Meggitt (1968). Like Wright (1921) and Ruhemann (1945) I have assumed discrete generation.

A comparison of the results of these studies is in the table below. The fact that my numbers are exactly 1/2 of those found by Wright and Ruhemann follows from s = (1/2)g in the minimal structure. Other examples of comparable systems may be found in the references already given, or also in Rossman and Rubel (1972), Dumont (1966), or Elkin (1950).

COMPARISON OF ISOMORPHIC STRUCTURES

System name according to Wright:	Wright (1921) "Size"	Ballonoff (1974) Structural Number "s"	Ruhemann (1945) "D"
Self-fertilization	1	not possible	--
Brother-sister	2	1	--
Double first cousin	4	2	4
(not named)	-	3	6
Quadruple second cousins	8	4	8
Octuple third cousins	16	8	16

Understanding why some existing systems chose one over another marriage rule may turn out to be a more difficult problem to solve than it was to raise (See also appendix to Ballonoff, 1982). But to intelligibly raise such questions as part of a more general theoretical framework is a necessary step for development of a <u>science</u> of anthropology.