CHAPTER 3 APPENDIX
ADDITIONAL DEMOGRAPHIC ASPECTS

The purpose of this appendix is to summarize additional comments on the foundations and techniques available for population planning on the basis of the present theory.

Basic Variables and Equations

It is useful to begin with a notational summary of variables. These are, with explanatory gloss:

n	:	a general representation of "family size" of which specific interpretations are n_s or n_i or $n(t)$,
n_s or n_i	:	the population average family size required for a population to be at zero growth and with stable social networks which follow rule with structural number s; or, when following a set $\{k\}$ of structural numbers of rules, with $i \varepsilon \{k\}$,
$n(t)$	:	the observed population value of n at time t,
p	:	the proportion of adults who are socially ascribed as active reproducers, i.e., are "married" by the rule with structural number s, or set of rules $\{k\}$,
p_s pr p_i	:	the proportion p which must be maintained for zero growth and network stability following a rule with structural number s or set of rules rule $\{k\}$ and $i \varepsilon \{k\}$,
$p(t)$	:	the observed population value of p at time t,
$v_i(t)$	:	the degree to which a population is using rule i at time t, where $\sum_i v_i(t) = 1$, $0 \leq v_i(t) \leq 1$,
$\bar{\mathbf{v}}(t)$	:	the vector of the $v_i(t)$ at time t,
T	:	the generation interval in years,
$r(t)$	:	the growth rate per year of the population in year t,
$r(t)T$	:	the growth rate of the population per generation measured in year t.

The basic equation which relates these variables is

$$e^{r(t)T} = \frac{1}{2} n(t)p(t) \tag{1}$$

where 1/2 is an adjustment for number of sexes.

If the population is following the set of structural numbers of rules $\{k\}$ to degrees $\bar{v}(t)$, then

$$p(t) = \sum_{i\varepsilon\{k\}} v_i(t)p_i$$

$$n(t) = \sum_{i\varepsilon\{k\}} v_i(t)n_i \qquad (2)$$

which assumes that the population is acting at equilibrium with respect to each rule to the degree to which it is following that rule, and that linear combinations are correct estimates of the combined rates (Ballonoff, 1976:119).

Note that equations (2) do not apply to a mixture of separate populations following different rules; they apply only to one inseparable population following several different rules. For example, the equations apply to a single cultural unit undergoing social change, or following some preference ordering by type of marriage, or allowing several different types of marriage while prohibiting others, etc. If the population itself has distinct cultural subunits, then the structural properties of each subunit would have to be analyzed separately. In this case distinct values must be computed for each cultural type. For example in a typical U.S. western state, the population averages of $n(t)$, $p(t)$, $r(t)T$ for the entire state must be computed as a set of values for each native American group, and for each non-native population. In particular, the state averages $n(t)$ and $p(t)$ will not predict the state value of $r(t)T$. Instead, the regional average $\hat{r}(t)T$ is the group average of the $r(t)T$ value for each subunit $u\varepsilon U$, the set of cultural units; or for $r(t)_u$ the theoretical growth unit for unit u is,

$$\hat{r}(t) = \sum_{u\varepsilon U} \frac{N_u(t)}{N(t)} r(t)_u \qquad (3)$$

where unit size $N_u(t)$ per generation in year t. Or, if the ratio $N_{u_1}(t)/N(t) = u$, where $N(t) = \sum_{u\varepsilon U} N_u(t)$, for unit 1, etc. then we can denote the vector of the u's by $\bar{u}$ and the vector of the $r(t)$'s by $\bar{r}(t)$, so that

$$\hat{r}(t) = \bar{u}(t)\bar{r}(t) \qquad (4)$$

Where there is only one rule, s, then behaving at zero growth with respect to the rule,

$$e^{r(t)T} = \frac{1}{2} n_s p_s \qquad (5)$$

However, if the population is following both $n(t) = n_s$ and $p(t) = p_s$ then by definition $r(t) = 0$ leads to:

$$n_s p_s = 2 \qquad (6)$$

which is also derived from completely different considerations in Ballonoff (1976, Chapter 6) as the relationship which must be obtained between n and p for populations following a rule with structural number s.

If more than one rule is in use, then from equations (1) and (2),

$$e^{r(t)T} = \frac{1}{2} \sum_{ij}\sum v_i(t)v_j(t)n_i p_j \tag{7}$$

Note that whenever $1 > v_i(t) > 0$ for some $i \epsilon k$, then there are at least two structural numbers in use, resulting in population growth, and that this occurs <u>in spite of the fact</u> that the n_i and p_i are taken as the equilibrium associated with use of rule i. (This can be seen from equation (13) later in this appendix). In other words, equation (7) specifically identifies <u>use of more than one social rule of marriage as itself a cause of population growth</u>.

Note that the assumptions summarized in the above paragraph may be restated as follows: divide the growth rate r(t) into a sum of $r_1(t)$ attributable to the effects of mixtures of rules, and $r_2(t)$ of all other effects, so that

$$e^{r(t)T} = e^{(r_1(t) + r_2(t))T} = e^{r_1(t)T} e^{r_2(t)T} \tag{8}$$

Equation (7) implies that $e^{r_1(t)T} > 1$ whenever more than one structural number is in use, while $e^{r_1(t)T} = 1$ whenever only one structural number is in use. This results from equation (6) applied to the given rule since acting according to the rule makes $n_i p_i = 2$. The assumptions say nothing about the other effects summarized by $r_2(t)$. In particular I make no assumption on whether or not there are linkages between types of causes, nor do I claim that $r_2(t) = 0$. However, I do compute the values of r(t) which result from considering only the effects of marriage rules alone. Therefore for present purposes let $r(t) = r_1(t)$.

Note also that, ignoring $r_2(t)$ effects, the existence of a regional steady-state distribution of $\bar{u}(t)$ requires either that each unit has $r(t)_u = 0$, i.e., each unit is following a single rule, or else that $r(t)_{u_1} = r(t)_{u_2} = \ldots$, etc., which restricts the possible values of $\bar{v}(t)_u$ and the specific sets of marriage rules possible for each unit $u \epsilon U$.

Or, in plain language, because of the linkages between p, n, population growth of particular cultural units, and marriage rules, it is not possible to create effective regional policy for either family size or proportion of reproducing adults without taking into account the particular ethnographic "cultural condition" of each cultural unit in the region. For example, policies aimed only at regulating "average family size" at some favored value run the serious risk of causing depopulation of some cultural units (rT could be negative!), increased population of other units, social or political irritations resulting from ignoring "cultural values" inherent in locally accepted marriage rules, and non-achievement of regional policy objectives. Note in the next section how such effects <u>of themselves</u> can be a cause, for example, of observed population growth in countries undergoing "modernization".

Theoretical Considerations

Effects of p

I note here a number of useful implications of the basic equations (1) through (7). First, note that for a given rule i, equation (6) implies that

$$n_i = \frac{2}{p_i} \tag{9}$$

so that the n values may all be eliminated from other equations, particularly equation (7). If we let $p_{ij} = \frac{1}{2}\left(\frac{p_i^2 + p_j^2}{p_i p_j}\right)$, and let P be the matrix of p_{ij}'s, then equation (7) can be written as

$$e^{r(t)T} = \bar{v}(t)\underline{P}\bar{v}(t)' \tag{10}$$

where $\bar{v}(t)$ is a row vector and $\bar{v}(t)'$, a column vector. Note that the linear dependence or independence of $\underline{P}$ depends upon the specific values of the p_{ij} which in turn depend on the p_i values.

In the two rule case, the determinant of $\underline{P}$ is equal to

$$\det(\underline{P}) = 1 - \frac{1}{4}\left(\frac{p_1^2 + p_2^2}{p_1 p_2}\right)^2 \tag{11}$$

which has the value 0 whenever $p_1 = p_2$, and is negative otherwise. This implies that the existence of (simply discovered) stead-state distributions depends on the inequality of the p_i values, since otherwise the matrix is linearly dependent.

It is possible to separate within equation (7) or (10) the effects of the $v_i(t)$ and the p_i on population growth. By expanding either form, it is found that each term in $v_i(t)^2$ has coefficient one, while each term in $v_i(t)v_j(t)$ for $i \neq k$ has coefficient $p_{ij} + p_{ji} = (p_i^2 + p_j^2)/p_i p_j$. If we let:

$$\begin{aligned} p &= \min(p_i, p_j) \\ p + x &= \max(p_i, p_j) \end{aligned} \tag{12}$$

then

$$p_{ij} + p_{ji} = \frac{(p + X)^2 + p^2}{(p + x)p} = 2 + \frac{x^2}{p^2 + px} \qquad (13)$$

Two conclusions follow immediately from (13). First, given any two rules i and j, there is a specific, computable contribution to growth resulting from the absolute value x of the difference between the p values of the two rules; this contribution is in a specific case weighted by the degree-of-use coefficients $v_i(t)$, while there is an additional contribution to growth of $v_i(t)^2$ for each rule in use. Second, the larger the value of this absolute difference x, the greater is the growth contribution of the interaction of the two rules.

This second point also has an interpretation in terms of the "complexity" of marriage rules, which can be illustrated using the following table of values of p_i for rules with structural numbers 1 = s = 2, 3, . . ., 10.

TABLE 1

VALUES OF p GIVEN s

s =	2	3	4	5	6	7	8	9	10
p_s =	1.0	1.0	.92	.82	.82	.73	.72	.63	.63

(from Ballonoff, 1976:91)

Now, to talk sensibly about "populations in equilibrium" one must specify exactly which equilibria one has in mind. For example, traditional population theory is concerned with a model in which population growth rates are associated with various age dependent variables such as fecundity or mortality. In that theory, the condition of zero growth is not always associated with a steady-state distribution of the age structure, or of steady-state age specific fecundity or mortality schedules. In the present theory there are likewise several different equilibria to worry about, which also may occur in association with non-zero population growth rates. The most critical of the possible equilibria for any given cultural unit u is clearly a steady-state distribution of $\bar{v}(t)$ which involves use of more than one rule. As already noted, this necessarily results in population growth, somewhat analogous to the case in which a steady-state age distribution results in population growth. Note in the table above that as s increases, p decreases and continues also to do so for s > 10). However, structural numbers in general are larger for more restrictive conditions on available kin-types: e.g., for prohibition of brothers and sisters only, s = 2; to prohibit all second cousins, s $\geq$ 9 may be required.

Thus, equation (13) implies that the greater the relative difference in "complexity" of rules, as measured by s, the greater the relative contribution to population growth resulting from simultaneous use of rules. This implies that, of itself, "severe" shifts in social practice have relatively greater effects on growth than do "milder" ones, for given values of $\bar{v}(t)$, during the period of transition.

The above conclusion should be modified in light of the following consideration of possible effects of "unmarriageability" on p and n values. Consider equation (6) for s = 2, . . ., 10, where p_s values are as given in the table above. Clearly if a proportion p_s of the population do "marry" by the social rules of the given system, then $1 - p_s$ do not marry by those rules. Particularly when p_s is close to 1, it is conceivable that "normal" biological conditions resulting in unmarriageability (such as gross physical defect, problems with health, etc.) may have an effect of depressing the maximum value that p_s could attain, with consequent effect on n_s as well.

Assume that there are two sources of unmarriageability: (1) the physical (and/or social) effects just noted; (2) statistical effects of population size. Call the sum of the proportionate effect of these two ε', and let $\varepsilon = 1 - \varepsilon'$ be the proportion of marriageables in the population. Then p_s is changed to a value εp_s, and one must correspondingly increase n_s to $(1/\varepsilon)n_s$, so that

$$\frac{1}{\varepsilon} n_s \cdot \varepsilon p_s = 2$$

still holds.

Table 2 shows the effect of $\varepsilon' = .01, .02, .03, .04, .05$, (i.e., of $\varepsilon = .99, .98, .97, .96, .95$) on p_s and n_s values for s = 2, . . ., 10. Note from Table 1 above that structural numbers may be grouped into the following sets whose p-values are equal to the second decimal place: (2,3), (4), (5,6), (7), (8), (9,10). The most obvious effect of ε' is to further blur the distinction between s = 7 and s = 8; for all other structural numbers there is no overlap in the ranges of n and p values added by effect of ε', while for s = 7 and s = 8 the already close values of p and n are now indistinguishable if the standard deviation in the estimate of ε' is as much as .02.

This conclusion must be modified if a different interpretation is given to p_s, namely that the proportion $1 - p_s$ will "already" allow for effects of ε', provided that $1 - p_s > \varepsilon'$ by some "meaningful" amount. In this case, Table 2 is only relevant for computing affects of ε' on s = 2, 3, and 4. Notice that the $(1/\varepsilon)n_s$ values thus computed for s = 2, s = 3 correspond closely to "zero growth" ideal average family size quoted by popular press advocates of population control.

Another major point is therefore that it is impossible to plan a "population control" program based on "average family size" (n) without simultaneously advocating

TABLE 2

EFFECT OF UNMARRIAGEABILITY SHOWING VALUES OF

$\frac{1}{\varepsilon} n_s$, εp_s FOR GIVEN VALUES OF ε

	$\varepsilon' = .01$		.02		.03		.04		.05	
	$\frac{1}{\varepsilon} n_s$	εp_s	$\frac{1}{\varepsilon} n_s$	εp_s	$\frac{1}{\varepsilon} n_s$	εp_s	$\frac{1}{\varepsilon} n_s$	εp_s	$\frac{1}{\varepsilon} n_s$	εp_s
s= 2	2.02	.990	2.040	.980	2.062	.970	2.083	.960	2.105	.950
3	2.02	.990	2.040	.980	2.062	.970	2.083	.960	2.105	.950
4	2.196	.911	2.218	.902	2.241	.892	2.264	.883	2.288	.874
5	2.464	.812	2.489	.804	2.514	.795	2.541	.787	2.567	.779
6	2.464	.812	2.489	.809	2.514	.795	2.541	.787	2.567	.779
7	2.730	.733	2.758	.725	2.786	.718	2.815	.710	2.845	.703
8	2.767	.723	2.796	.715	2.824	.708	2.853	.701	2.884	.694
9	2.970	.673	3.001	.666	3.032	.659	3.063	.653	3.096	.646
10	2.970	.673	3.001	.666	3.062	.659	3.063	.653	3.096	.646

control of the proportion of "married" persons (p) at a particular value corresponding to the stated family size (A point almost never raised in the popular press or political advocacy literature!). This point is also true if a specified amount of population growth (or decline!) is allowed, since in the general form

$$e^{rT} = \frac{1}{2} np \qquad (5')$$

assuming T is a fixed parameter, specifying any two of r, n or p necessarily fixes the value of the third. T can also become a planning tool, provided the age structure, meaning fecundity and mortality rates, are subject to manipulation, and/or that enforceable laws may exist on age of marriage or age of birth of (first) child for females. (In general it appears to the author that moral practices of most systems prefer to not manipulate T by such direct legal methods and instead to accept its "naturally" determined values. Manipulation of p is almost as unacceptable in most cultures as well; while couples may chafe at having their number of offspring limited, they may be even less happy if told by authorities to reproduce even when they do not so desire.)

Summary of Causes of Growth

As is evident from previous sections, growth of a population may have several possible causes which can be identified on the basis of the present theory. For example, if the structural number is s, then if either or both the observed n or observed p exceed n_s or p_s respectively with the other equal to the theoretical value, or if the empirical product $n.p > 2$ then population growth occurs*, and is in act proportional to $\ln(\frac{1}{2}\, p.n)$.

Thus far I have used four different concepts of "family size" (more or less explicitly). I therefore list these different concepts in the following and introduce notation to distinguish them.

Notation for value of:	n	p	Description
(a)	n_s	p_s	the equilibrium values associated with following a rule of structural number s (independent of population size)
(b)	n(t)	p(t)	as defined, the values of n and p considered as linear combinations at time t of the n_s and p_s values of all rules in use at that time by the given cultural unit (independent of population size)
(c)	n_{sex}	ρ	the values required in a population of a particular size to maintain equilibrium taking into account fluctuations in sex ratio, associated with structural number of a given rule (minimal-size dependent)
(d)	n_I n_{II} etc.	p_I p_{II} etc.	values of n and p at each level I, II, etc. of a multi-layered lineage organized system

The values denoted as n_{sex} were inherent in the discussion of the section on unmarriageability, but I here make them more explicit. The values n_I, n_{II}, p_I, p_{II}, etc. were laid out in Ballonoff 1983. As used there, n_I, p_I correspond to n_s, p_s for systems where the marriage rule is stated in terms of lineage rules rather than in terms of kin-relations. Where both rules are present, such as section-systems where kin-type and clan are equivalent then restrictions such as $n_s = n_I$, $p_s = p_I$ are further required. Values subscripted II, III, etc. refer to population statistics of higher level groupings (number of families per lineage, lineages per clan, etc.).

Table 3 shows relative sizes of these various quantities for s from 2 to 15 using values of n_s and n_{sex}, p_s, and $2/n_{sex} = \rho$ computed in Ballonoff (1976a) chapter 5

* Remember again that these "growth" values could be negative, resulting in decline. Some discussion of this appears later in this chapter 3 appendix, and in Ballonoff 1982 or 1983.

and 6. Column (1) shows the average family size needed to maintain a population using a rule with structural numbers, considering only probabilistic fluctuations in sex ratio (from chapter 5, Ballonoff, 1976). Denote this quantity as n_{sex}. Column (2) shows p_s, the value required by rule s; (3) shows $n_{sex}p_s$; column (4) shows ρ.

TABLE 3

COMPARISON OF VARIOUS VALUES OF n AND OF p AND THEIR PRODUCTS

	(1)	(2)	(3)	(4)
s	n_{sex}	p_s	$n_{sex}p_s$	ρ
2	4.0	1.0	4.0	.50
3	2.65	1.0	2.65	.75
4	2.55	.92	2.34	.78
5	2.45	.82	2.01	.82
6	2.40	.82	1.97	.83
7	2.35	.74	1.74	.85
8	2.32	.73	1.69	.86
9	2.29	.68	1.56	.87
10	2.27	.68	1.56	.88
11	2.25	.66	1.49	.88
12	2.23	.63	1.40	.90
13	2.22	.63	1.40	.90
14	2.21	.61	1.35	.90
15	2.20	.60	1.34	.91

There are several lessons to be learned from Table 3. First, if a planning program specialized on "zero growth values" that were associated with "normal statistical fluctuations in sex ratio" in populations of minimal size (such as rural towns) then it would prescribe values of column (1) for the minimal sized units of systems with structural numbers indicated. If this system were behaving according to its marriage rule, it would achieve the value p_s of column (2). But such policies do not take into account p_s or social conditions. The joint effect of such "zero growth" policy and social practice is shown in column (3): rapid growth for systems with small structural numbers, ($n_{sex}p_s > 2$), and depopulation for systems with larger structural numbers or more complex lineage systems, ($n_{sex}p_s < 2$). Thus, the political objections raised by some "underdeveloped peoples" or "underprivileged classes" to "zero growth planning" may well be justified by the present theory! Zero growth planning which does not account for marriage rates p can have almost every possible outcome except zero growth, which it will only attain as a matter of luck.

Column (4) shows the values which p would have to take in order for "zero growth" planning at column (1) values to be successful -- i.e., to result in zero growth. In general these are not the same as the values, from table 1 of this appendix, required for simultaneous cultural stability and zero growth.

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Thus, the mathematical theory of anthropology produces, somewhat even to the surprise of the author, an argument showing a direct relationship between more popular philosophies of population planning, and political opposition to the practice of family size based on those philosophies, which opposition has an evident technical foundation when planning is viewed in a more complete context.

Growth Rate Risks in Lineage Systems

Lineage organized systems are essentially those which group offspring into families (level I), families into "lineages" (level II), lineages into (perhaps) "clans" (level III), and so forth. Given a particular number of units to be maintained at a particular level, it is possible to predict upper and lower bounds for the number of units to be maintained at each lower level.

A population could remain stable with a fixed number of local units and fixed number of units at each level of organization, if it remained always within these bounds; however, it could also undergo "budding" by amoeba-like division each time it reaches the upper bound, provided the upper bound is roughly twice the lower bound. It is conceivable that a system could carry out such a process while still remaining within the bounds of n_I values due to the following fact: as population size increase, for any given system with given number of layers, n_I increases and p_I decreases (See Table 2 of Chapter 3).

At the upper and lower bound, using superscript "u" to denote upper bound and "ℓ" lower bound values,

$$n_I^u p_I^u = 2$$

$$n_I^\ell p_I^\ell = 2$$

both hold, but also $n_I^u > n_I^\ell$, $p_I^u < p_I^\ell$, so that in particular

$$n_I^u p_I^\ell > 2 \ .$$

Therefore a system could remain consistent with its rules of lineage organization, while maintaining a constant growth rate up to the maximum amount

$$\ell n\left(\frac{1}{2}\ n_I^u p_I^\ell\right) \ .$$

It is possible to view the upper and lower bound values as the boundaries of a stochastic sampling process. In this case,

$\ell n(\frac{1}{2}\, n^u p^\ell)$

is the maximum population growth rate risk attainable in a socially stable system of the given organization, and

$\ell n(\frac{1}{2}\, n^\ell p^u)$

is the maximum population decline rate risk for a socially stable system for the same description. The simultaneous interpretation of these values as rates and as risked rates is vital. Note that the two risks of increase and of decline are in general equal for the particular system; therefore the various tables list only the positive or "growth" risk.

As a concrete example (using Table 6.1 from Ballonoff, 1976; pages 107-111) if the upper bound size is 200 per generation then $n_I^u = 3.98$, $p_I^u = .50$; if the lower bound size is 100 per generation then $p_I^\ell = .57$, $n_I^\ell = 3.54$, so $\ell n(\frac{1}{2} \cdot (.57) \cdot (3.98)) = .126$, or 12.6% growth per generation. Depending on the generation interval, this could represent a yearly growth rate of as little as .5% or nearly as much as 2.0%. Tables of these risks are found in the appendix to Ballonoff (1983).

An important realization is that a system could be consistent with its rules while declining in total group size. However, declining populations will either soon disappear or soon stop declining. As a practical observational and historical prediction, it is only those systems which consistently grow which are likely to be observed. This fact, together with the fact that upper and lower bounds exist for each rule and/or lineage organized system, leads to the observation that particular growth rates may be characteristic of particular organizations. Further, when a system uses both a lineage organized and a structural-numbered marriage rule, it may only exist if both rules have compatible growth rates since the population itself has only one growth rate. Therefore, the likelihood of simultaneous observation of more than one cultural rule in a single system may be determined by comparison of growth rates possible under different types of rules.

It is therefore possible to compute purely from theoretical considerations which cultural rules are most likely to be observed, and furthermore it is possible to predict what will be the range of population size(s) at which each rule will likely be observed. This was done for marriage rules in the appendix to Ballonoff (1982) for sequence I, II, and III systems respectively, by use of prediction down. Values in that paper were based on Table 6.1 of Ballonoff (1976) for L and N values, while the maximum values of r(t)T were computed as above. The table was therefore also useful for computing the sizes of hierarchies observable in particular systems with the indicated sequence III systems, with up to 7 units at the top. The table is limited by the practical fact that Table 6.1 of Ballonoff (1976) did not include values beyond N = 242 due to limits of the combinational table of Andrews (1965).

However, an enterprizing reader could apply the linear approximations published in Ballonoff (1976).

The growth rate risks computed above are the maximum attainable while preserving the social rules in a "pure" system, i.e., one not in transition between two or more rules. Carefully note that a single system using two rules (such as structurally-numbered rule and a lineage-organized rule) may be growing at a positive rate, but still be a "pure system" in that all individuals are equally subject to the same rules. Also, note that I have assumed in this discussion that the rules for males and females are either the same or have the same sequence and structural numbers. Where the rules differ, then equations of Ballonoff (1976, chapter 7) will apply. In such cases one of the following must be true: the system has a highly disproportionate sex-ratio and furthermore is short-lived since this disproportion will continue to grow; the system imposes severe methods of regulating sex-ratio external to the marriage rules; the population growth rates of the rules are similar for both sexes. Modifications involving several of these possibilities at once and leading for example to cyclic variations in sex ratio can be easily imagined.

Comparing Growth Risks

The text has thus far identified at least two distinct sources of population growth inherent in the existence of marriage rules: that which is possible in "pure systems" acting within their compatible bounds as found from characteristic of the Stirling Number; and that which results from change in rules. It is useful to compare the orders of magnitude of the amount of growth resulting from each of these sources. Table 4 shows the theoretical growth rates per generation which would be generated by transitions between each part of structural numbers listed. The table lists only values for "half" of a transition since the rates are completely symmetrical in the degree of use coefficients. The highest growth rates occur, as was computed earlier, when the p values involved in the transition differ the most. It requires a transition between s = 2 and s = 15 (or so) to attain growth rates per generation as high or higher than are observable with any of the lineage organizations summarized in the appendix tables of Ballonoff (1982).

The next table presents a set of values for growth per generation which wold be associated with following a structural numbered rule. However, it recognizes that the values of n and of p published in previous tables ignored the "prediction down" or "fan" effects associated with use of the Stirling Number (previous values assumed only that the maximum possibility density was the predictor of n and p value). Table 5 computes the maximum compatible growth rates, treating the L value which leads to the initial computation of n and p values for structural numbers as if it were the top of a sequence I prediction down. This does not imply that structural-numbered systems are also lineage systems; however, it recognizes that the logic used earlier leads to a numerical computation for structural numbered systems identical to prediction down for a sequence I system.

TABLE 4

GROWTH RATES PER GENERATION FOR SYSTEMS IN

TRANSITION BETWEEN STRUCTURAL NUMBERS INDICATED

$\bar{v}(t)$ =	(1.0,0.0)	(.10,.90)	(.20,.80)	(.30,.70)	(.40,.60)	(.50,.50)
(s=2, s=4)	0	.0006	.0011	.0014	.0016	.0017
(s=2, s=5) (s=2, s=6)	0	.0035	.0063	.0082	.0094	.0098
(s=2, s=7) (s=2, s=8)	0	00.82	.0145	.0190	.0217	.0026
(s=2, s=9) (s=2, s=10)	0	.0135	.0238	.0311	.0355	.0370
(s=2, s=15)	0	.0440	.0770	.0999	.1133	.1178
(s=4, s=9) (s=4, s=10)	0	.0082	.0146	.0191	.0219	.0228

Evaluation of Tables 4 and 5 lead to some quite interesting interpretations. Consider graphs 1 and 2 below. Graph 1 shows the maximum growth rate risks per generation, graphed against the L of the highest level, by sequence number. Graph 2 shows the maximum per generation growth rate risks graphed against N at lowest level of organization. The most obvious feature of the two graphs is that the order of sequence I, II and III are completely reversed between the graphs: Sequence I has uniformly the highest growth rate risks for number of units at the highest level of organization compared to other sequences, but the lowest growth rate risks if population size per generation of lowest levels of organization are considered. Detailed discussion of these results can be found in the appendix to Ballonoff (1982).

A second interpretation follows from the first: if the computed population sizes are total per generation group sizes, they can also be the per generation sizes at which individual isolates with relatively low interaction rates between them, can stand a very high chance of survival even if complete isolation occurs. Therefore, the minimal numbers are not predictions of actual per generation population size, but predictions of the multiples in which actual sizes must occur. This same point can be made from another perspective, as developed in Ballonoff (1976) chapter 4, or in Ballonoff (1976b) on social network phenomena. As is widely recognized in anthropological literature on "exchange systems", particular cultures have characteristic sets of social networks. It is characteristic of minimal structures

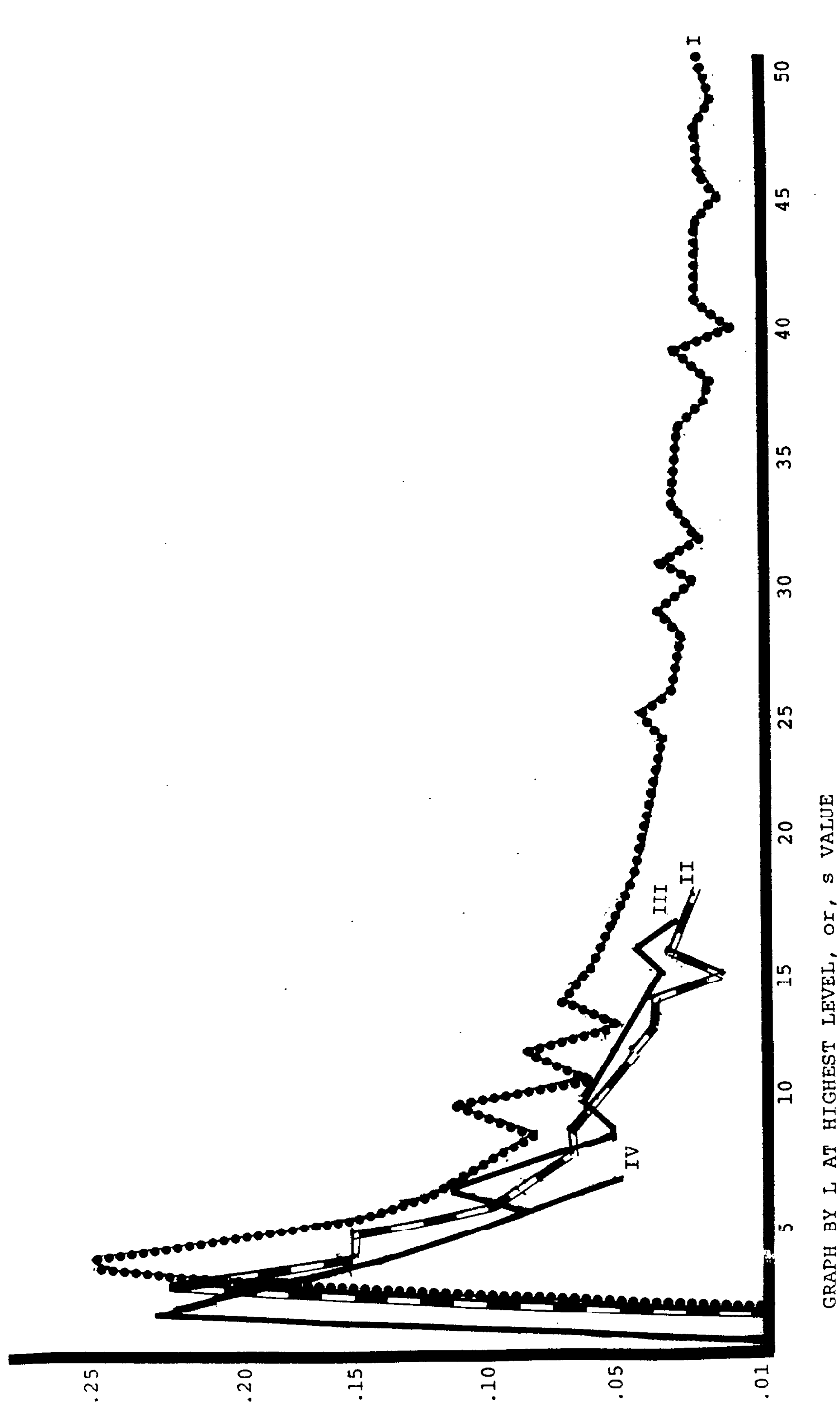
TABLE 5

GROWTH RATES CHARACTERISTIC OF

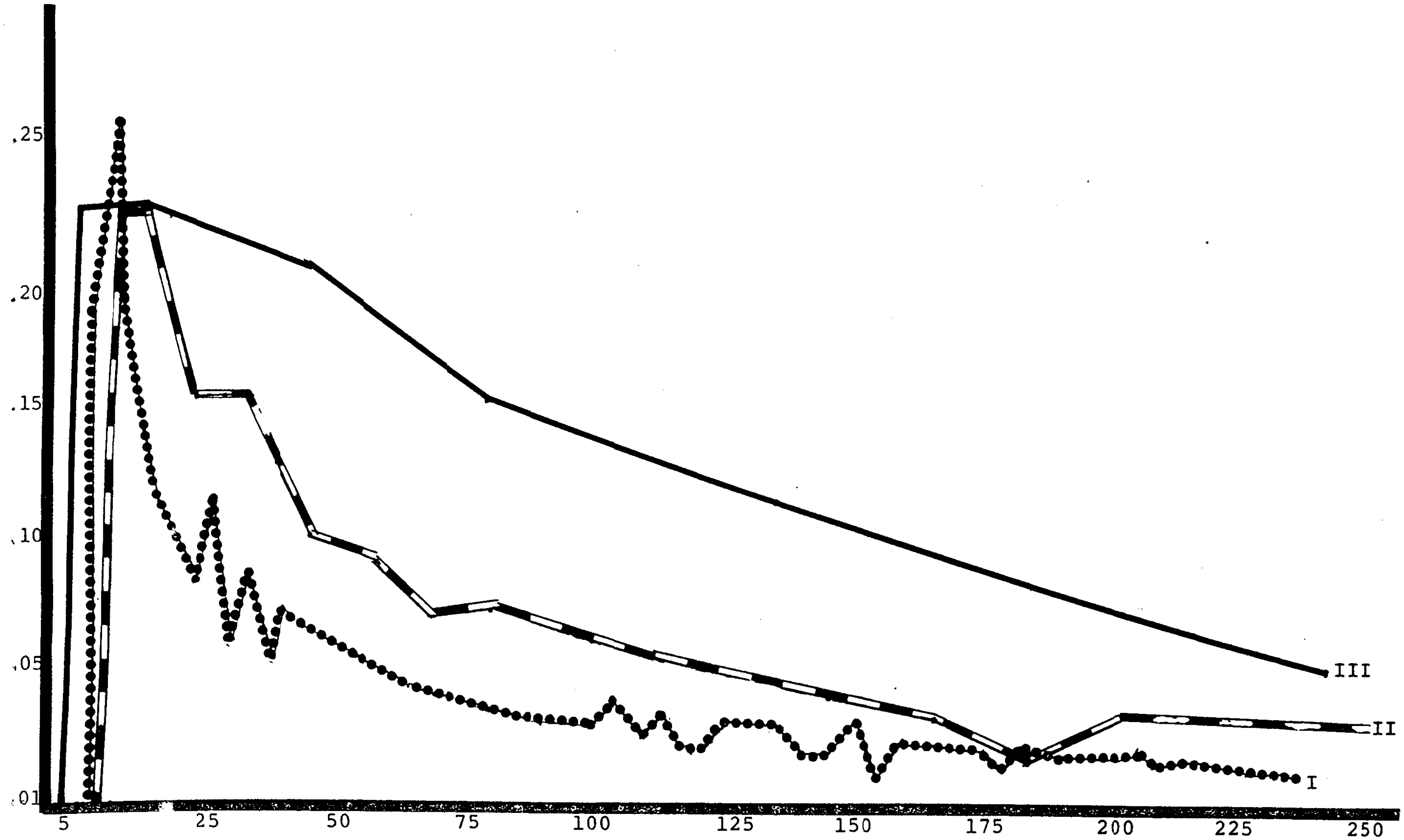
STRUCTURAL-NUMBERED SYSTEMS IN SEQUENCE I

S	N	Corresponding L, Seq. I	r(t)T
2	4	2	0
3	8	4	.182
4	13	6	.143
5	17	7	.117
6	22	9	.087
7	27	10	.113
8	33	12	.089
9	38	13	.054
10	44	15	.068
11	51	17	.058
12	57	18	.054
13	64	20	.047
14	69	21	.042
15	76	23	.039
16	83	25	.046
17	96	28	.030
18	99	28	.030

that they preserve the possibility of forming networks. Furthermore the group-theoretical properties of minimal structures require that where the networks are not inclusive of the entire population, then at minimum sizes they partition the population into subsets, of the sizes of products of the prime factors of the population size. Thus, use of either survival probability criteria, or arguments from combinatorial group theory, require interpretation of "minimal" population sizes as the "unit weight" of which larger aggregates must be comprised. The "unit weight" interpretation is also that which governed construction of the theory discussed in Ballonoff (1982, 1982b and 1983).


GRAPH BY L AT HIGHEST LEVEL, or, s VALUE
.01
.05
.10
.15
.20
.25
5
10
15
20
25
30
35
40
45
50
I
II
III
IV

GRAPH 2



GRAPH BY N AT LOWEST LEVEL

Comparative Summary

With this background, it is possible and useful to present a table summarizing foundations and characteristics of the known aspects of the theory. Four versions of the theory are summarized as rows A, B, C and D below.* Entries of line A are summaries of the properties of the theory resulting from considerations in Ballonoff (1976, Chapters 4 and 6) of marriage rules based on structural numbered rules; line B entries are based primarily on "purely random" or probabilistic considerations which ignore marriage rules (from Ballonoff 1976, Chapter 5); line C shows properties of the theory which result from study of lineage organized systems per Ballonoff (1982, 1982b and 1983); line D summarizes the rule-independent model studied in Ballonoff (1976), Chapter 5, using aspects of line A and B models.

The topic "marriage theory", that is, the theory proposed by this book, refers to lines A and C. Theories of lines B and D were used to compare marriage theory results to policy effects of ignoring marriage theory. The theories of lines B and D more closely approximate existing "normal" population theory concepts. The major thesis of this text may be simply summarized: ignore marriage theory (lines A and C) at your peril.

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* Row A refers to the theory of structural numbers (i.e., that of n_S and p_S statistics) which relates to kin-based marriage rules; Row B relates to n_{sex}-based computations for local units; Row C relates to estimates for lineage organized systems (n_I, p_I estimates); Row D discusses the theory based on n_{sex} and ρ estimates. Note that although the pairs (A, C) and (B, D) are more similar in computation, (A, B) and (C, D) are more similar in their effective areas of application.

ORIGIN OF THEORY: POPULATION STATISTICS
A. Theory of structural numbers, plus $s \leq N - N_o$; $N_o = N\ (1 - \frac{1}{N})^F$ and Stirling Number of Second Kind.
B. After finding minimal values for rules, compute a Binomial model of excess males or females to find the value of n_{sex} for the population.
C. Theory of lineage organizations, based on Stirling Number of the Second Kind.
D. Estimate n_{sex}, from empirical population size using binomial distribution model. The compute $\rho = \frac{2}{n_{sex}}$.

	Abbreviated Definition	Imputed Value n	Inputed Value of ρ	Predict Minimal Sizes	Predict Actual Sizes	Do estimates depend on population size?	Probability Linked Estimates	Predict Growth
A	Kin-based rules	n_s	ρ_s	yes	only if minimal	only near minimum size	some theory exists	possible
B	sex ratio only considered as minimum for rule	n_s	ignores ρ_{sex}	uses minimum to get estimate	no	complete size dependence	is a probability based statistic	possible
C	Lineage organized systems as layer i	n_i	ρ_i	yes	yes	aggregate estimates possible; estimates depend on unit size	theory does not exist--probably branching processes	possible
D	empirical pop size used to predict n, ρ independent of rule using n_{sex}, ρ_{sex}	n_{sex}	$\rho = \frac{2}{n_{sex}}$ whenever $n_{sex} < n_s$	can compare to minimal sizes & infer compatible rules	derived from actual sizes	size dependent	yes--if sex linked--otherwise not known	inherent

	NATURE OF POPULATION PROCESS
A	Stable at minimal sizes. Small growth for probability reasons just above minimal sizes. Stable at large aggregates if "minimal behaving" sub-units. Growth if aggregates if each act above minimal values.
B	Use for "first insight" into sources of growth resulting from equilibrium behavior.
C	"Budding Processes" when expanding. Upper and lower bounds for local groups found by theory. May be stable, depending on local use of upper and lower bound value and whether system locally exogeneous or not.
D	"Family Size Driven Budding Process." Population growth (or decline) depends on choice of n_s or n_{sex} and of p_s or ρ in making social network. Model associated with weak or absent lineage organization.