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A MATHEMATICAL THEORY OF CULTURE

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PREFACE

The present work summarizes the foundations for and meaning of an essentially new theory of cultural form, to which the present author has been the main contributor. The work is not intended as a general mathematical theory of all aspects of culture. Certain topics necessary to such a study have thus been omitted, particularly those theories of linguistics and of kinship terminologies which have received extensive algebraic treatment elsewhere.

On the other hand, I do discuss what I have herein glossed as "systems theory" or "thermodynamics" or sometimes "statistical mechanics". These concepts have so dominated social sciences in the last forty years that it often seems work can not be published (indeed, is not considered "science") without at least a pretense that the work is part of the "systems" tradition. It is particularly towards these concepts that I direct the extended quote from Peierls at the start of the Forward.

Thus, I have several specific purposes. One purpose is to explain the meaning of results. From that point of view, I assume little mathematical knowledge; I would hope that even a non-mathematical reader can gain from reading particularly Chapters 1, 2, most of 3, and certainly Chapter 5. I have specifically attempted to put the "worst mathematics" into the appendix to Chapter 3, or in the major Appendices I and II. In any event, the theory of minimal structures itself is mathematically rather simple.

This simplicity notwithstanding much of the discussion deals with questions more akin to mathematical philosophy than to anthropology or culture as one usually thinks of the subject. I hope interested readers will find this discussion of value both for understanding what I have already completed, and for following their own thoughts in similar directions. Readers familiar with standard population biology mathematics will realize that I am treating problems quite different from their usual concerns. Those who approach this book looking for familiar mathematics applied in familiar ways will be frustrated. Readers who accept the subject as defined in Chapters 1 and 2 will hopefully find my treatment of it constructive.

An important non-mathematical section of work which I present here for the first time is Chapter 5, "Evolutionary Implications". This chapter is essentially a response to arguments that minimal structure theory (also called here "marriage theory") seems divorced from biological reality. For example, authors of the "sociobiology" tradition particularly believe they have already solved the central problem of this text. If so, their solution is wrong. Chapter 5 argues that the biological evolution of intelligence, and of the capacity for culture, is instead profoundly compatible with the mathematical results available from marriage theory. The biological uniqueness of individuals, well established in biochemistry and an important point of moral philosophy, is also embodied in the mathematical foundations of the present work. One need not violate this presumption to derive a theory of evolution of cultural form nor of the origin of cognition generally, and of intelligence in particular.

Thus, the overall text has a fairly simple organization. The main chapters develop the most substantive theory, marriage theory and its implications. Once completing the conceptual development in Chapter 3, Chapters 4 and 5 give technical support to the technique and implications of marriage theory. This (together with the appendices) concludes the technical work. Chapter 6 indulges more in speculation and philosophy. It does not add to the theory, but may help explain why I constructed it as I did.

Certain technical results are useful or necessary to parts of my argument. I have therefore reprinted, as Appendix I, the text of my original paper on the theory of minimal structures, in the exact form (except for references) which I presented to the 1973 Ethnological Congress. A slightly modified version later appeared as Chapter 4 of my 1976 book, but as the concepts are so important to other work, publication here is justified.

Appendix II contains, evidently for the first time in other than limited circulation, a paper of Schadach. A number of critics of my work have the impression that I somehow assumed that the Stirling Number of the Second Kind was essential to population theory. As such, this would certainly be an off-the-wall assumption, in need of justification. However, that is not what happened. At first, I used the statistic because it worked for its application, and because it clearly fit the results I had otherwise obtained by numerical techniques such as noted in Chapter 4. Then I discovered that the framework was but one of several sorts studied by Schadach in his classifications of mappings and their combinatorial properties.

While Schadach has as no apparent purpose the development of a social theory, his paper proves that a structure of the sort used in my algebraic description of marriage rules and minimal structures, requires that the Stirling Number of the Second Kind be associated with the distributional properties of such mappings. Because this statistic can be interpreted as distributing unique objects into non-unique locations (or in this context, individuals into roles), the uniqueness of individuals is actually derived as a conclusion from the present theory. The fact that this conclusion is also required by biological reality is strong evidence that the concepts proposed here are at least steps in the right direction toward a science of culture. The moral implication, however, is also quite satisfying.

Note that I have not simply "applied mathematics" to some preconceived notion, say, of cultural stability, and then proceeded to lift handy theorems to get "results". With the single exception of the Stirling Number result (where luck alone found me a copy of an unpublished result that was exactly pertinent) I have had to construct almost the entire theory from scratch. As I note for example in parts of Chapter 3, direct application of "standard" mathematical techniques frequently at best leads to study of a sideline to the main theory. The problem has been not only to construct a theory which was sufficiently rigorous that it was, in fact, mathematical, but also to interpret what that construction meant, empirically and technically.

This interpretation problem is especially tricky for the Stirling Number results, since marriage theory is actually two theories: a theory of kin-based rules have structural numbers, and a theory of lineage organized systems that do not. It turns out that structural number theory may be considered in certain ways as a low order example of lineage theory. Lineage theory is perhaps easier to explain, but many readers of this book will no doubt think of "marriage systems" as dealing primarily with kinship related structures, whether or not connected to lineage systems.

Both theories use the Stirling Number of the Second Kind, but in slightly different ways, and with importantly different qualitative results. For example, structural number theory predicts certain population-size-independent values, except that it also predicts minimum sizes for populations using kin-based rules. Linage theory predicts all results in connection with the total population size associated with them, including of course minimum sizes. Also, empirically as well as in development of theory, it is often hard to distinguish a complex kin-based system (with high structural number) from a lineage organized system; both can and do simultaneously occur in real systems.

It thus took some time to realize that two different but closely related theories were required, and to describe their properties. Although structural number theory is a bit more complex, it also lends itself more easily to mathematically tractable results by analytical means. Thus, most of what has been done "analytically" in this work is based on structural number results. Most work on lineage theory is highly numerical. In many cases, however, the qualitative result, if not the precise numerical formulation, which comes from structural number theory will also apply to a similarly phrased problem in lineage theory. Thus, while I hope I have clearly separated when I am using one or the other theory, I am not too worried about any apparent ambiguities that may remain.

A note of my use, and non-use, of footnotes. In general, I have avoided these. An essential argument belongs in the main text; a supporting but more technical argument belongs in one of the appendices. Much supporting detail is already published and is referenced by a statement of source. However, in Chapter 5, my arguments on evolution of intelligence are not previously published. Since I believe many of my statements in the main argument do require more detailed support, in that section as well as in the Forward, I have used footnotes to avoid distracting from the flow of thought. This does not mean I believe the footnotes are of less interest than the main text. Footnotes follow the chapter in which they appear.

The present book is self-contained. Useful tables or results which were derived or presented earlier are referred to when appropriate, and tables reproduced (with sources given) when needed. The most frequent references are to my 1976 monograph, from which Chapters 3 through 7 have some application here). The present book however is devoted almost entirely to developments since the earlier work was written, or (as in Chapters 1 and 2 here) to background useful to either book. The intellectual heart of the present work is Appendix I, and Chapter 3 and its appendix.

I also wish to acknowledge the hard work of my typist, Georgia Porter, on the many retypes of this manuscript.

FOREWARD

IMPLICATIONS OF A MATHEMATICAL THEORY OF SOCIAL ANTHROPOLOGY

"In claiming that biology is not likely to be a branch of the present physics, I do not wish to imply that life can in some mysterious way evade the laws of physics. I believe that the situation is comparable to the problem of electricity and magnetism as it appeared before, and even during, the time of Maxwell. Physics then was mechanics, and to explain a phenomenon meant to find its mechanism. Even Maxwell tried hard to back his field equations by mechanical models. He understood only later that electric and magnetic fields were important basic concepts in physics, not contained in the concepts of mechanics, but of course not in contradiction to the concepts of mechanics -- and that physics had to be enriched by adding them. It is at least possible, and to me probable, that similar new concepts have to be added to our present physical ones before an adequate description of life is possible. Whether the thus enlarged discipline should still be called physics is a semantic question."

Rudolph Peierls, Surprizes in Theoretical Physics
Princeton University Press (1979), page 34.

A recent two-part paper [1] and others [2] have developed the underlying mathematics for a theory of social anthropology, and also have applied it to ethnographic data [3] with success. But this theory is not only new chronologically, it is also new in that it differs in important ways from pre-existing mathematical or quantitative social science. Theses differences are important in much the same way that the differences of quantum mechanics from statistical mechanics are important to physical theory: they imply or derive from different philosophical outlooks and mathematical frameworks, yet predict inter-related aspects of the same or similar phenomena. Neither replaces the other, though at times the correspondence between the theories requires more than passing analysis.

In the social or evolutionary sciences, mathematical theories exist in several areas that appear to be similar to the domain of social anthropology. For example, demography has a detailed theory of the relationship of age-structured dependent fertility and mortality schedules to average family sizes, certain steady state population characteristics, and the like [4]. Evolutionary theory has a well developed mathematical theory of population genetics, including of "inbreeding" effects, regular systems of mating, and like topics [5]. Economics theory has quite well developed technologies for analysis of efficient decision making, optimal flow in organized economic systems, use of resources, and the like [6]. Social networks theories have developed a great deal of algebra for analysis of relational systems [7]. And, general systems theory has amalgamated similar approaches into a single framework [8].

Thus, it is fair to ask why and how the new theory of social anthropology differs from these other subject theories. The essential technical reason for this distinction is found in the papers of footnotes [1] and [2]. Essentially, the distributional statistic of the underlying density function for mathematical social anthropology is not the same density function used in statistical mechanics. Statistical mechanics (and all of the other literature referenced in notes [4], [5] and [6]) relies on the Stirling Number of the First Kind (SNFK) to generate the underlying density function. But the theory of social anthropology relies on the Stirling Number of the Second Kind (SNSK) for the underlying density function [9]. This SNSK density function is required by the algebraic properties of the maps developed in the references [1], and [2] to describe the operations of social rules of marriage.

While the new theory is in no sense a "quantum social theory" and is not a derivation from physical quantum theory, the parallel is still useful since traditional quantitative social theory has strong ties to statistical mechanics (or, thermodynamical) methods. In fact these ties are often taken as the basis for a "general systems theory" [8] for systems research applications. This discussion does not challenge that the inter-disciplinary lessens of systems theory are well founded. For example, mathematical economics and mathematical biology have gained a great deal by mutual exchange of ideas, and in particular by ideas of "efficiency" closely related to thermodynamics [10]. Very similar statistical mechanical methods underlie more modern studies of equilibrium behavior of economies [11] and of evolutionary behavior of genetically evolving populations [12] and are widespread in studies of microeconomics, population genetics, demography, and engineering economy type analyses [13].

What makes all these applications capable of similar mathematical treatments is that they share statistical assumptions similar to those used to construct the density functions of statistical mechanics. In a typical problem, statistical mechanics considers all of the possible ways that gas molecules could be distributed in a room or box. It assumes essentially that all the molecules are identical and that all of the locations are unique. By looking at all of the combinations of ways of placing molecules, the conclusion is that there are a few solutions where all the molecules congregate into one corner, but the overwhelming mass of possibilities has molecules spread throughout the box more or less evenly. Thus, the possibility function or counting functions of the underlying combinational mathematics translates directly into a probability statement, with very good confidence, on the probable distributions of gas in a box. Such theories are very well refined and accurate in applications such as the flow of heat in various materials, least cost electrical dispatch on a power system, or the probability of certain outcomes in genetic selection or hybridization work such as summarized in references above. In all cases, there is an underlying physical system with similar combinational properties.

That is why the discovery summarized in references of notes [1] and [2] that a different combinational density function is required by social anthropology is so important. This discovery arose from two simultaneous pieces of information. Shortly after the algebraic mappings required to describe social networks were developed (particularly as published in works in note [2] but chronologically in 1971) the author came into possession of papers by Andrew (per note [9]). These papers describe the relationship between classifications of systems of mappings and the combinational functions induced on partitions of sets by those classifications of mappings. Thus, it was only necessary to decide which classification of mappings included those of references [2] in order to discover, using Andrew's work, the correct combinational foundation. Remarkably, while many of the references in note [7] are better developed as to certain relational properties, none of them asked the question on combinational foundations, and therefore none realized the profound possibilities that may result.

The fact that the combinational density function is different in this than for other applications raises correspondence problems in interpreting results. Essentially the correspondence problems result from the fact that the two kinds of theories (i.e., statistical mechanical and quantum) ask, and answer, different kinds of questions about the same population. Statistical mechanical results relate to the existence of equilibrium and non-equilibrium states of system meeting the statistical assumptions of SNFK -- i.e., identical objects into unique locations. Thus population genetics studies distributions of a limited number of types of genes into individuals; economics the production of identical or similar products sent to market by different firms. Demography looks largely at stationarity properties of SNFK density functions with an SNFK probability shift operator. Statistical mechanics and general systems methods based on that or information theory use SNFK statistics, often based directly on the exact mathematical methods devised for genetics or thermodynamics theories.

Social anthropology, however, much like quantum mechanics, instead asks questions on the rules of cultural bonding, and on the effects of particular bonding rules on measures of systems following such rules. This subtle difference in the asking and answering of questions does not mean that one or another of the types of theories is wrong, nor even that their results are incompatible.

For example, social anthropology per notes [1] and [2] does not ask nor answer questions on age-structured fertility and mortality; nor does demography answer questions on effects of marriage and kinship rules on population measures. If in a particular ethnographic application the two give distinct answers, this may well imply that there is a great deal left to be learned about social dynamics. However, one does not toss out statistical mechanics for example, merely because it does not of itself fully describe the structure of the helium atom.

Thus, population genetics analyzes the consequence of strict biological adherence to following specified systems of mating, i.e., of biological transmission of genes from parent to offspring. Social anthropology, however, studies the effects of different systems of marriage, i.e., of establishing certain cultural or legal bonds among people, which cultural bonds may or may not biologically determine the physical transmission of genes from biological parents to biological offspring. As in mathematics itself, such subtle distinctions have profound importance in analytical results.

It is therefore not surprising that correspondence questions may arise when using the two different kinds of analysis on the same population. It is indeed likely in both the demographic and the genetic examples above that many apparent discrepancies with cultural theory results arise not from the theories themselves but from subtle but important semantic differences in interpretation of the theory. The difference between biological mating and cultural marriage is one example. In demographic studies, the classical models study "growth" in the sense of change in total numbers, while social anthropology studies "growth" in terms of change in the relative ability of the population to maintain the cultural rules [14]. These two notions are not identical, but are related [15].

Thus, the existence of a new mathematical theory for social anthropology does not insinuate that pre-existing theories of other subjects are wrong for their applications. Instead, it implies that additional theory for previously unaccessible problems is possible. This in turn implies that the definitional concepts of "general systems theory" may have to be broadened if the general system concept is to be indeed general. However, it also implies that a more effective theory can result - one which will include cultural analysis in a truly scientific sense.

FOOTNOTES TO FOREWARD

[1] See Ballonoff (1982a,b).
[2] See Ballonoff (1976, 1983).
[3] See items in notes [1] and [2] and also Ballonoff (1973).
[4] The mathematics of demography as a study of age-structured populations is now a well developed science, with a history can be traced to 1662 in works of John Graunt. The modern theory is particularly based on Lotka (1907), Sharpe and Lotka (1911), Lewis (1942) and Leslie (1945). An excellent summary of the modern theory is Keyfitz (1968).
[5] Population genetics has one and perhaps several well developed theories, also founded in the mathematics of probability. A comprehensive summary is in Crow and Kimura (1970). A work which particularly illustrates the "thermodynamic" foundations of population genetics is Kimura (1964). Works showing how these probability processes relate to structural mating include Karlin (1968) and particularly Wright (1921).
[6] There is no secret to the similarity of methods in economics and thermodynamics. The result has been summarized in work such as Intriligator (1971). A good illustration of the use of economic analysis on a physical dynamic problem is Kirchmayer (1958, 1959).
[7] The literature on social networks could not be summarized in a short volume. Examples include White (1970), Coleman (1964), works in Ballonoff (1974a,b) and very many other works.
[8] A "general systems" study uses ideas from information theory and statistical mechanism as the keys to analysis. An excellent example is Schmeikal (1980).
[9] See works by Andrew (1975a,b).
[10] Works such as by Lotka (1925) freely draw on economics ideas; while economic and social statistics, especially correlation studies, draw on genetics models such as developed in the 1920s and 1930s by Wright (1968) initially for breeding studies.
[11] See Intriligator (1971) as an example.
[12] See Kimura (1964) as a well developed example.
[13] See note [6].
[14] See in particular the appendix to Ballonoff, 1982a, or discussions in text of Ballonoff, 1983.
[15] An equation describing the relationship is found in Ballonoff, 1982b.