

MATHEMATICAL MODELS
OF SOCIAL AND
COGNITIVE STRUCTURES
Contributions to the Mathematical
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STATISTICAL THEORY OF MARRIAGE STRUCTURES

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After a short statement of how marriage structures may be characterized by a single number, or set of related numbers, the paper explores three approaches to generating population statistics a priori from a knowledge only of these structurally determined numbers. A comparison of these three approaches is offered, and several areas in which further progress may be made are suggested.

BACKGROUND

It is readily apparent to readers or writers of ethnographies that populations are often subdivided into numerous biologically effective units which carry out reproductive functions of the system. The various units are either geographic or culturally defined so that they interchange marriage partners according to seemingly fixed rules or patterns, dependent on the existence of certain numbers of the units in question. A good example of this is found in the classic Australian systems, for which fundamental numbers have long been a main concern of anthropological theory (see Ruheman 1945, or Livingstone 1959).

Less widely recognized, but nonetheless true, is that almost any marriage system has associated with it certain characteristic numbers, which are the minimum number of matings, biological families, and persons of each sex necessary to maintain the claimed rule. The theory

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of these structural numbers has been described elsewhere (Ballonoff 1970, 1973; Ballonoff and Duchamp 1971; Lorrain 1973, and this volume), but the essential characteristics for the present purpose are these. First, for nearly any system in which monogamy is the mode or for which the marriage rule may be described on a genealogical skeleton which assumes monogamy, a unique structural number exists. Second, it can be proven rigorously that the usual assumption of anthropologists of one male and one female per family (or two and two in different families under parallel cousin rules) is adequate for efficient description of the minimal case. Third, the structural number of the system is equal to the number of marriages per generation in the minimal genealogical skeleton. This notion of minimality and its characteristics are important in the arguments that follow.

PURPOSE OF THE PAPER

I can now state my purpose as relating the structural number of a system to the demography of that system. By demography I mean population statistics such as total size per generation, average family size, number of families, and fertility of females. In particular, I shall be interested in these statistics at or close to the minimum numbers—but, as we shall see, these can also be relevant for much larger population sizes. As my first evidence, I point out that if the number of monogamous matings is s , then the minimal population size must be $2s$, since there is one male and one female per monogamous pair in the diagram which depicts the system. For good illustrations of such minimal systems, I suggest the paper by Livingstone mentioned above, or the illustrations found in Harris (1971). Note that for random mating the minimum number of matings is one; for sibling exclusion, the number is two; for sibling exclusion and either patrilineal or matrilineal exclusion of first cousins, the number is three; for bilateral first cousin exclusion, the number is four; etc.

To gain greater insight into the demographic implications, three different models will be discussed and compared. These are: probability of system survival as functions of structural numbers and population statistics; analytical requisites on statistics as determined by structural numbers; stochastic modeling of population distributions

dependent upon structural numbers. I will present a rudimentary model from each approach. A more extended development of both structural and statistical aspects is found in Ballonoff (1974).

The empirical importance of the results may be described as follows. For prescriptive rules in a closed (i.e., locally endogamous) system, the theory should give close predictions of the empirical statistics and system behavior. For locally exogamous prescriptive rules, the predictions may also be lower bounds, since large system effects can lower average values needed for stability. For systems with indeterminate structural numbers, no predictions are offered. It should be noticed that the ability to adequately describe a system limits theoretical predictive power, and not population size per se.

SURVIVAL PROBABILITIES

My first argument (which is incidentally also the weakest conceptually, but easiest to formulate) calculates the survival probabilities of minimal systems. Apart from the social theoretical implications of the argument itself, I present this material in the hope that persons skilled in reliability engineering may recognize the strong similarities to their own work and thereby improve upon my arguments.

I begin with a provable statement which will be taken on faith here, to wit: a very large number of the conceivable marriage systems can be represented by sets of pairs of sibships in sister exchange. For even-numbered structural numbers, we are interested in the probability that each of all of the $\frac{1}{2}s = k$ pairings survive, where s is the minimal structural number, and k is the number of pairings of sibships whose members exchange sisters in the minimal system.

For each of the sibships, if the family size is 2, then the possible sex distributions are (m,m) , (m,f) and (f,f) , where m and f respectively mean "male" and "female". If the proportions of males and females are shown by \underline{m} and \underline{f} , since everyone is one or the other, $\underline{m} + \underline{f} = 1$. By a simple binomial expansion we find the expected frequencies of the above sex distributions as $(\underline{m} + \underline{f})^2 = \underline{m}^2 + 2\underline{m}\underline{f} + \underline{f}^2$. Since this gives the distributions for each family, the possible distributions for each pairing of families are given by squaring as in the following table:

$$\begin{array}{l}
 \frac{(m,m)}{m^4} \quad \frac{(n,f)}{2n^3 f} \quad \frac{(f,f)}{2m^2 f^2} \\
 \frac{(m,f)}{2mf} \quad \frac{(n,f)}{2n^2 f} \quad \frac{(f,f)}{2mf^3} \\
 \frac{(f,f)}{f^2} \quad \frac{(n,f)}{2mf^2} \quad \frac{(f,f)}{f^4}
 \end{array}$$

From this point, it is possible to construct a large number of quite different survival probabilities, each of which follows from different assumptions on the interpretations of "survival," and "survival of what." I will here calculate two different probabilities, each of which is consistent with the rigorous conditions of the second paragraph. This will demonstrate the method and provide some very interesting results to be interpreted. Others may then make different calculations as they wish.

First, since the absolute minimal conditions for most systems require an equal number of males and females, only back diagonal entries are fully viable for all systems. Likewise, the two corner entries f^4 and m^4 are totally nonviable, while the remaining four entries with one or another term cubed are partially viable. To avoid complicating demographic arguments, and to maintain consistence in my assumption of strict monogamy, I shall treat all partially viable terms indifferently even though in fact those distributions with more females than males may actually be better off reproductively.

I can now generalize slightly by assuming the average family size (with zero variance) is \bar{n} . Using this assumption, the nonviable portions become $\frac{2}{3}\bar{n}$ and $\frac{1}{3}\bar{n}$. If I use V for the proportion of fully nonviables, then

$$V = \frac{2}{3}\bar{n} + \frac{1}{3}\bar{n}.$$

Or, if one makes the sensible assumption $\bar{m} = \bar{f} = \frac{1}{2}\bar{n}$,

$$(1) \quad V = 2\left(\frac{1}{2}\right)^2 \bar{n} = \left(\frac{1}{2}\right)^2 \bar{n}^{-1}.$$

Similarly, there are four partially viable terms whose sum V_p , with equal sex ratios, is:

$$(2) \quad V_p = \left(\frac{1}{2}\right)^2 \bar{n}^{-2}.$$

I will now define two different survival probabilities. S_1 will be the survival if only fully nonviables are removed, while S_2 removed both fully and partially nonviables, so that

$$\begin{aligned}
 (3) \quad S_1 &= (1 - V)^k \\
 S_2 &= (1 - V_p)^k
 \end{aligned}$$

are the survival probabilities for the entire system of k pairs under these assumptions. Equations (3) in turn imply that the replacement number per individual under each assumption must be $1/S_1$ and $1/S_2$ respectively; likewise, the replacement per mating pair, hence "average family size", must be $2/S_1$ or $2/S_2$. Note that k in the exponent is (only) justified by assuming each pair reproduces the appropriate ratios independently of whether or not any other pair does so.

Computation of $S_1, S_2, 2/S_1, 2/S_2$ for $k = 1, 2, \dots, 20$ (structural numbers 2, 4, $\dots, 40$) for family sizes 2 through 10 in increments of .2 was carried out by computer. The table below for $k = 4$ ($s = 8$) shows a typical pattern: S_1 starts at low values, but quickly, usually by $n \leq 4.0$, reaches a 90 percent or higher S_1 survival; while S_2 survival may require astonishingly large replacement per person at small values, $1/S_2$ also drops as \bar{n} increases.

Results of these computations will be used later in the paper, but note here that this model essentially assumes zero variance. Thus these probability calculations may be considered overestimates of the true values with Poisson or binomial variance due to the increase in uncertainty.

STOCHASTIC MODEL

As a second model, consider a society maintaining L arbitrary families, where "family" simply means biological reproduction unit at the level above the individual. Clearly, $s \leq L$, for any system with an

interpretable marriage rule, and intuitively one suspects the strict inequality must hold. If there are N unique individuals born per generation, and all and only the L families are kept, then these N must be distributed into L such that no L is empty. The standard computation for the number of ways N distinct objects can be placed into L indistinguishable cells with no cell empty is the Stirling Number of the Second Kind, $S(N,L)$, which has been calculated in a table (Andrews and von Foerster 1965).

For any N value, $1 \leq L \leq N$ is conceivable; but which of these shall actually occur, and how are they related to structural numbers? If the problem is posed as meeting unknown circumstances with the greatest variability of individual traits, then this maximum variability should occur with the L value for a given N which has the highest number of combinations of possible placements. Calling this "max L " for "maximum L value, given N ," I used the table to find these values.

Treating max L as the expectation of number of lineages or basic families over a large number of trials of size N , this statistic has several implications. First dropping the "max" from max L , but remembering that $L = \max L$, $N/L =$ average family size. Next, since there must at equilibrium be the equivalent of one female reproducing fully per reproduction unit, $L/N =$ the proportion which must be female reproducers from the whole population; with $\underline{m} = \underline{f} = 1/2$, then $2L/N$ gives the proportion of the female population which must be engaged in full-time reproduction, defined as each producing the N/L offspring per generation while $1 - \frac{2L}{N}$ produce no offspring. These values are summarized in Table II, for $1 \leq N \leq 50$. N and L are from the published table, while all other entries are simple calculations from these.

Since the published table stops at $242 = N$, it is interesting to attempt to fit a curve to large values of (N,L) pairs from the original table. Unfortunately, using the standard UCLA Biomed polynomial regression program, a single regression does not suitably fit the entire data set; most give polynomials whose graphs are obviously focused in the wrong direction outside the original data set. (For

TABLE I. Probability of Survival at Various Family Sizes for: $k = 4$, $s = 8$

\bar{n}	$1/S_1$	S_1	$1/S_2$	S_2
2.0	1.706	0.586	50.567	0.019
2.2	1.489	0.671	17.568	0.056
2.4	1.347	0.742	8.620	0.115
2.6	1.250	0.799	5.186	0.192
2.8	1.183	0.844	3.563	0.280
3.0	1.135	0.880	2.684	0.372
3.2	1.100	0.908	2.161	0.462
3.4	1.075	0.930	1.828	0.547
3.6	1.056	0.946	1.604	0.623
3.8	1.042	0.959	1.449	0.689
4.0	1.031	0.969	1.338	0.747
4.2	1.024	0.976	1.257	0.795
4.4	1.018	0.982	1.197	0.835
4.6	1.013	0.986	1.151	0.868
4.8	1.010	0.989	1.117	0.895
5.0	1.007	0.992	1.090	0.916
5.2	1.005	0.994	1.070	0.934
5.4	1.004	0.995	1.054	0.948
5.6	1.003	0.996	1.042	0.959
5.8	1.002	0.997	1.033	0.967
6.0	1.002	0.998	1.025	0.974

example, $L > N$ is impossible but is "predicted" using the complete data base.) However, using only the last fifty values, $L = 9.80 + .19N$ is the only regression which appears valid for large N , although for values outside any of our interest it is also wrong. (See Rennie and Dobson [1969] for an asymptotic approximation at very large N .) Using this estimator, the handy asymptotic ratio $\frac{L}{N} = .20$ occurs for large N . This implies that the largest family size ever required by a social system is $n = 5$, but it also implies that the family size increases with N , while simultaneously $\frac{2L}{N}$ decreases with N , from > 1.0 at the smallest values, to $.40$ at large values. This corresponds to the notion that in small societies $\frac{2L}{N} \sim 1$, so that every woman is almost always fully engaged in reproduction, while in large societies only .40 of the women need be so engaged, provided each turns out five offspring. (Notice we are still using a zero-variance model.)

These results will be further interpreted below. Also note that for $N \leq 8$, $\frac{2L}{N} > 1$ for all cases except these pairs: $(N = 4, L = 2)$, $(n = 6, L = 3)$, $(N = 8, L = 4)$, and, therefore, no other pairs at these small values are physically possible in equilibrium in this model, since they require more reproduction capacity than exists in the population. This is consistent with purely structural arguments of Ballonoff (1973): these are the only cases where $s = L = \frac{1}{2}N$, a required structural condition where: actual populations approach the theoretical minimal. Finally, the existence of local maxima of $\frac{2L}{N}$ after each increment of L and local minima before each increment should not be ignored: this small discontinuity implies that as populations seek to minimize reproductive burden, they will also increase it according to the law: two steps forward, one step back!

Consider still a third method of attack. Suppose there are N families per generation, and that there are F females. Let the chance that a particular birth occurs in a particular family be $\frac{1}{N}$, and hence $\frac{N-1}{N}$ that it does not occur in that family. Note that if N_0 families have no female offspring, this may be regarded as a "proportion of failure" as in Part I; hence we must have

$$(4) \quad s \leq N - N_0.$$

TABLE II. Predictions of L and Its Properties from $L = \text{Max } K$

N	L	L/N	2L/N	N/L
1	1	1.0000	2.0000	1.0000
2	2	1.0000	2.0000	1.0000
3	3	1.0000	2.0000	1.0000
4	4	1.0000	2.0000	1.0000
5	5	1.0000	2.0000	1.0000
6	6	1.0000	2.0000	1.0000
7	7	1.0000	2.0000	1.0000
8	8	1.0000	2.0000	1.0000
9	9	1.0000	2.0000	1.0000
10	10	1.0000	2.0000	1.0000
11	11	0.9091	1.8182	1.1000
12	12	0.8333	1.6667	1.2000
13	13	0.7692	1.5385	1.3000
14	14	0.7143	1.4286	1.4000
15	15	0.6667	1.3333	1.5000
16	16	0.6250	1.2500	1.6000
17	17	0.5882	1.1765	1.7000
18	18	0.5556	1.1111	1.8000
19	19	0.5263	1.0526	1.9000
20	20	0.5000	1.0000	2.0000
21	21	0.4762	0.9524	2.1000
22	22	0.4545	0.9091	2.2000
23	23	0.4348	0.8710	2.3000
24	24	0.4167	0.8333	2.4000
25	25	0.4000	0.8000	2.5000
26	26	0.3846	0.7692	2.6000
27	27	0.3704	0.7407	2.7000
28	28	0.3571	0.7143	2.8000
29	29	0.3448	0.6977	2.8333
30	30	0.3333	0.6667	2.8571
31	31	0.3226	0.6467	2.8857
32	32	0.3125	0.6250	2.9167
33	33	0.3030	0.6061	2.9500
34	34	0.2941	0.5882	2.9857
35	35	0.2857	0.5714	3.0238
36	36	0.2778	0.5556	3.0643
37	37	0.2703	0.5405	3.1073
38	38	0.2632	0.5263	3.1526
39	39	0.2564	0.5128	3.2000
40	40	0.2500	0.5000	3.2500
41	41	0.2439	0.4878	3.3024
42	42	0.2381	0.4762	3.3571
43	43	0.2326	0.4651	3.4138
44	44	0.2273	0.4545	3.4727
45	45	0.2222	0.4444	3.5333
46	46	0.2174	0.4348	3.5952
47	47	0.2128	0.4256	3.6586
48	48	0.2083	0.4167	3.7234
49	49	0.2040	0.4081	3.7896
50	50	0.2000	0.4000	3.8571
51	51	0.1961	0.3922	3.9260
52	52	0.1923	0.3853	4.0000
53	53	0.1887	0.3792	4.0750
54	54	0.1852	0.3737	4.1510
55	55	0.1818	0.3688	4.2283
56	56	0.1786	0.3643	4.3069
57	57	0.1756	0.3602	4.3867
58	58	0.1727	0.3564	4.4677
59	59	0.1700	0.3529	4.5498
60	60	0.1675	0.3497	4.6330
61	61	0.1651	0.3467	4.7173
62	62	0.1628	0.3439	4.8027
63	63	0.1606	0.3413	4.8892
64	64	0.1586	0.3389	4.9767
65	65	0.1567	0.3367	5.0652
66	66	0.1549	0.3347	5.1547
67	67	0.1532	0.3328	5.2452
68	68	0.1517	0.3310	5.3367
69	69	0.1503	0.3293	5.4292
70	70	0.1490	0.3277	5.5227
71	71	0.1478	0.3262	5.6172
72	72	0.1467	0.3248	5.7127
73	73	0.1457	0.3235	5.8092
74	74	0.1448	0.3223	5.9067
75	75	0.1440	0.3212	6.0052
76	76	0.1432	0.3202	6.1047
77	77	0.1425	0.3193	6.2052
78	78	0.1418	0.3184	6.3067
79	79	0.1412	0.3176	6.4092
80	80	0.1406	0.3168	6.5127
81	81	0.1401	0.3161	6.6172
82	82	0.1396	0.3154	6.7227
83	83	0.1391	0.3148	6.8292
84	84	0.1386	0.3142	6.9367
85	85	0.1382	0.3137	7.0452
86	86	0.1378	0.3132	7.1547
87	87	0.1374	0.3127	7.2652
88	88	0.1370	0.3123	7.3767
89	89	0.1367	0.3119	7.4892
90	90	0.1364	0.3115	7.6027
91	91	0.1361	0.3111	7.7172
92	92	0.1358	0.3108	7.8327
93	93	0.1355	0.3105	7.9492
94	94	0.1352	0.3102	8.0667
95	95	0.1350	0.3100	8.1852
96	96	0.1347	0.3097	8.3047
97	97	0.1345	0.3095	8.4252
98	98	0.1343	0.3093	8.5467
99	99	0.1341	0.3091	8.6692
100	100	0.1340	0.3090	8.7927

Noticing that $\binom{N-1}{N} F$ is the probability that with F births, none is female, then, assuming independence,

$$(5) \quad N_0 = \binom{N-1}{N} F$$

is the expected proportion of families which do not reproduce females. This is a permissible calculation since $\binom{N-1}{N} F$ may also be seen as the F = i term in the binomial expansion

$$\sum_{i=0}^F \binom{F}{i} p^i q^{F-i} = (p+q)^F$$

$$\text{with } p = \binom{N-1}{N} F, \quad q = 1 - p.$$

It is now possible to derive one direct relationship between s, F, and N, at the equilibrium values of F = L (that is, one fully reproducing female per biological unit). This is done by placing an equality in (4) and solving for F, which gives

$$(6) \quad F = \frac{\ln\left(\frac{N-s}{N}\right)}{\ln\left(\frac{N-1}{N}\right)}$$

Note that (6) graphs with the equilibrium values of each s on the 45° line through the origin, and with F = s, N = s as vertical and horizontal asymptotes.

Treating s and N as inputs, by numerical simulation the equilibrium values for 1 ≤ s ≤ 31 were found, and are reported for s ≤ 18 in the column "N = F" of Table III. This same simulation discovered that for all points F = L (where F_e shows this equilibrium point):

$$\frac{dF}{dN} \Big|_{F_e} = -.72.$$

Only for s ≤ 4 did this derivative stray from the .71 - .73 range, and then only by ~ .01. This constancy is to me unexpected, and I would like to suggest that if correct, this may become a very useful statis-

tic for predicting the possibility and direction of structural change: between marriage rules of particular systems. Such an argument would, however, require a more global model, where the particular derivative was an important threshold value.

Another result appeared from the simulation: F_e is closely approximated for the simulated range by a simple linear equation:

$$F_e = .23 + .56s$$

which may be verified by examining the "F_e - s" column in Table IIIa. This means that the result of simple substitutions into the estimation equations for max L from N given above are quadratic and linear predictions of N and M from s.

DISCUSSION OF COMPUTED VALUES

The various statistics described in earlier sections may now be compared, along with two other statistics of interest. Table III summarizes the important values. In column 1 of Table IIIe are values of s = 1 to s = 18 representing almost any marriage system known empirically. Column 3 shows the equilibrium values F_e = L found from the analytic model by numerical simulation, assuming one equivalent reproducing female per fully reproducing family, with column 2 showing the discovered N at this point. In column 4 are the required proportions of reproducing females for stability under the stochastic model, at the point F_e, while column 5 gives the N/F_e = \bar{n}_e average family size for equilibrium under the stochastic model. Column 6 gives average family size for equilibrium where only accidental imbalances in sex ratio are considered (based on an unpublished simulation). Finally, column 7 gives the F_e - s values used above, and column 9 gives values of t, an empirical constant to be discussed below.

Table IIIb shows the calculated probabilities S₁, S₂, and family sizes 2/S₁, 2/S₂. These are presented here since the values at equilibrium (S₁ ~ .82), and (S₂ ~ .25) appear to be independent of the structural number, a result I find even more surprising than the constancy of the derivative mentioned above. It should be an interesting

TABLE IIIa. Values of s and Associated Statistics

(1) s	(2) N	(3) $L=F_e$	(4) $2L/N$	(5) \bar{n}_e	(6) \bar{n}_{sex}	(7) $F_e - s$	(8) r
2	4	2.8	1.00	2.00	4.00	0.8	.30
3	8	4.4	1.00	2.00	2.65	1.4	.406
4	13	6.0	0.92	2.12	2.55	2.0	.450
5	17	7.6	0.82	2.43	2.45	2.6	.450
6	22	9.2	0.82	2.44	2.40	3.2	.459
7	27	10.8	0.74	2.70	2.35	3.8	.447
8	33	12.3	0.73	2.75	2.32	4.3	.453
9	38	13.9	0.68	2.92	2.29	4.9	.438
10	44	15.5	0.66	2.93	2.27	5.5	.436
11	51	17.1	0.66	3.00	2.25	6.1	.425
12	57	18.7	0.63	3.17	2.23	6.7	.417
13	64	20.3	0.63	3.20	2.22	7.3	.411
14	69	21.8	0.61	3.29	2.21	7.8	-
15	76	23.4	0.60	3.30	2.20	8.4	-
16	83	25.0	0.60	3.32	2.19	9.1	-
17	96	26.6	0.58	3.42	2.17	9.6	-
18	99	28.2	0.58	3.42	2.17	10.2	-

Note: Values in this table were done by numerical simulation by techniques discussed in the paper. Values entered as "-" were either not within the capacity of the computer, or not amenable to estimation by the chosen technique.

TABLE IIIb. Values of Various Survival Probabilities at the Equilibrium Population Sizes of Table IIIa

s	S_1	$2/S_1$	S_2	$2/S_2$
2	.874	1.29	.375	5.33
4	.800	1.50	.200	10.00
6	.810	1.46	.230	8.70
8	.845	1.36	.280	7.12
10	.830	1.40	.290	6.70
12	.840	1.38	.280	7.14
14	.845	1.36	.259	7.72
16	.825	1.42	.214	9.34

challenge to an anthropologically curious mathematician to redo this argument in order to predict this constancy of survival probability at minimum values from the same global arguments that predict constant: derivative $\frac{dF}{dN}$ at minimal equilibrium. That they are related is doubtless, since both are directly dependent upon the proportion of females in the population.

This stability of the probability model is not reflected in the stochastic approximation of the analytic equilibrium (column 5), which has an average family size increasing from 2.00 to 3.42 as s goes to 18, and goes above 5.00 for very large s . However, there is a relationship between the probability model and the stochastic simulation found by comparing the family sizes of the stochastic-analytic model to random-mating, of column 6, for given s . These values graph as shown in Figure 1, and the point of intersection is $2.44 = N/F_e = \bar{n}_e = \bar{n}_r$.

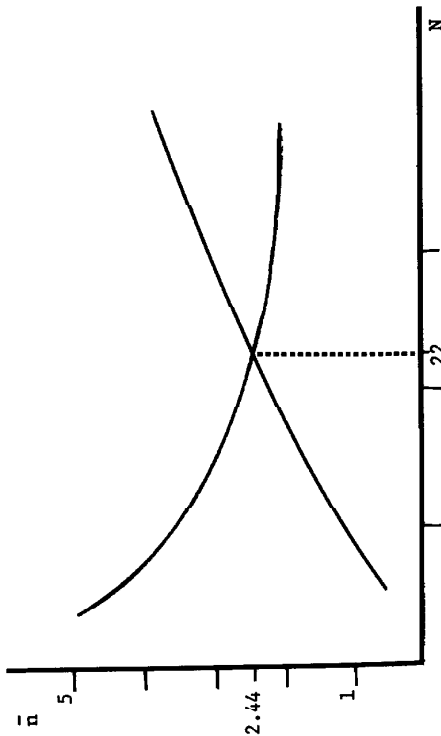


FIGURE 1. Intersection of random and structured mating equilibrium points. (Not to scale)

Figure 2 then illustrates the implied empirical prediction for the range of possible marriage structures as N increases, under the assumption that a system will be as efficient as possible with its investment in population size. Note that for generation cohort size

$N = 22$, the system will attempt to attain the highest structural number possible; but since all such systems are underpopulated relative to sex ratio requirement, these systems will be highly unstable. Once past $N = 22$, it is no longer necessary to force family size up and improve stability by increasing the structural number. In the range $22 \leq N \leq 100$, most any system with structural numbers $s \leq 6$ might be found, but the larger numbers are certainly not compatible with $\bar{n} \sim 2.11$ for $N \sim 100$. Indeed, only $s = 4$ is compatible with maintaining sufficient population size to account for sex ratio proportion for medium to large N .

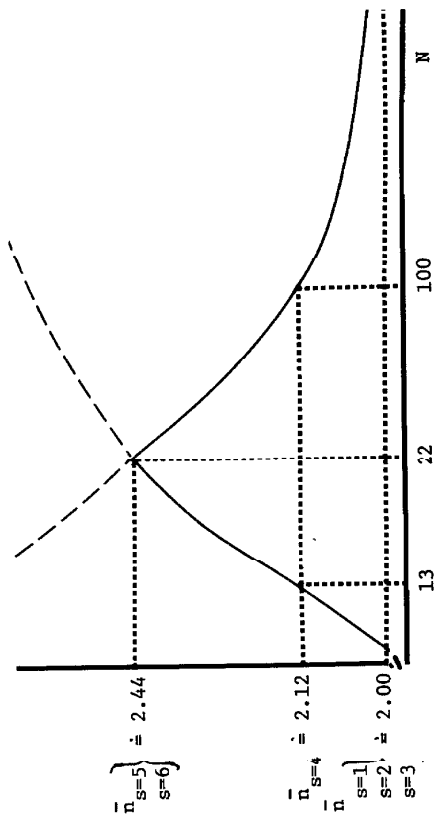


FIGURE 2. "Least effort" structure prediction for values of population sizes shown by solid line. Equilibrium sizes for various structural numbers shown on vertical axis. Note that $\bar{n} = 2.00$ is a lower asymptote under random mating.

Thus, in metropolitan societies, one would expect to find rules no more complicated than first-cousin exclusion, while in small societies, more complicated systems are necessary for survival. In very small systems, only a limited number of combinations of numbers of families with numbers of offspring are physically possible; hence one would expect the most rigid rules, such as Australian systems with group theoretical properties, to occur in smallish populations. Such rigid rules, as seen from Table II, can only have 4, 6, 8, and perhaps 10

as the number of basic reproduction units, and these must be regulated by structural numbers $s \leq 6$. For all systems, the proportion of females engaged in reproduction is fairly high, being above .8, even for large systems using $s = 4$.

But there are other and equally startling implications. If a system is operating at equilibrium with the average size of interacting unit s being small, and the addition of modern transportation, etc., suddenly makes greater migration possible, then with no change in marriage practices or no modification in health techniques, the fact that the relevant N for sex equilibrium is now larger means that \bar{n} for sex equilibrium is smaller, and sudden growth will result from the old marriage practices. The model also implies that average family size is not regulated by total system size (for example, U.S. population total is irrelevant to the equilibrium family size). Rather, the neighborhood size is the determinant of local average family size, and the national average is simply an average of these local unit sizes. This average will always be greater, by a meaningfully large amount, than the equilibrium predicted for the whole system as a single unit. Note that studies of propinquity of residence in several American cities appear to show that relatively small neighborhoods do exist (see Davie and Reeves 1939; Bossard 1932; Kerckhoff 1956; Abrams 1943). It would be useful to compare the rates of flow between such neighborhoods using the statistics developed by Lombardi, in this volume.

As a final note I point out, using the empirical constant r found in Table IIIa, column 8, that

$$s = \int -rNdN = N_0(N - N_0) - \frac{1}{2}N^2.$$

I conclude by posing the problem of the empirical meaning of r . In this expression, the "integral" is actually fictitious, and the expression $N_0(N - N_0)$ is certainly not a constant. In fact, the discovery of this equation was largely accidental, and its form here deliberately suggestive if incorrect. The form is intriguing, since the final expression closely resembles growth equations often quoted in demography texts.

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